



Computational Methods for Model Predictive Control New Opportunities for Computational Scientists

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Computational Methods for Model Predictive Control

New Opportunities for Computational Scientists

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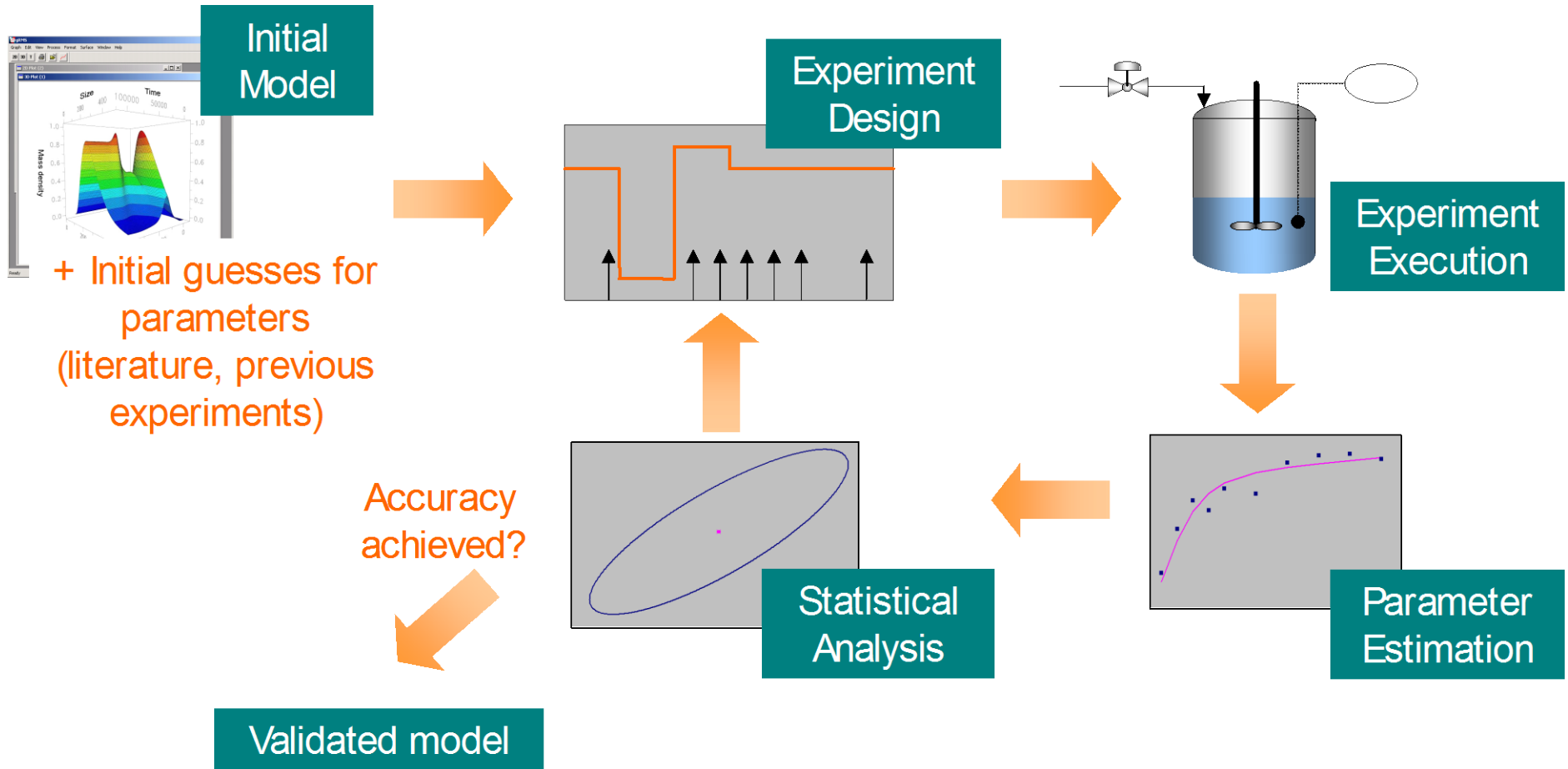
PARA 2012, Finlandia Hall, Helsinki, Finland
June 10-13, 2012

Introduction

- Model Predictive Control
 - Optimal Experimental Design
 - Parameter Estimation
 - State Estimation
 - Regulation
- Constrained Optimization of Dynamical Systems
- Case Studies
 - The Artificial Pancreas
 - Closed-Loop Oil Reservoir Management
 - Smart Energy Systems

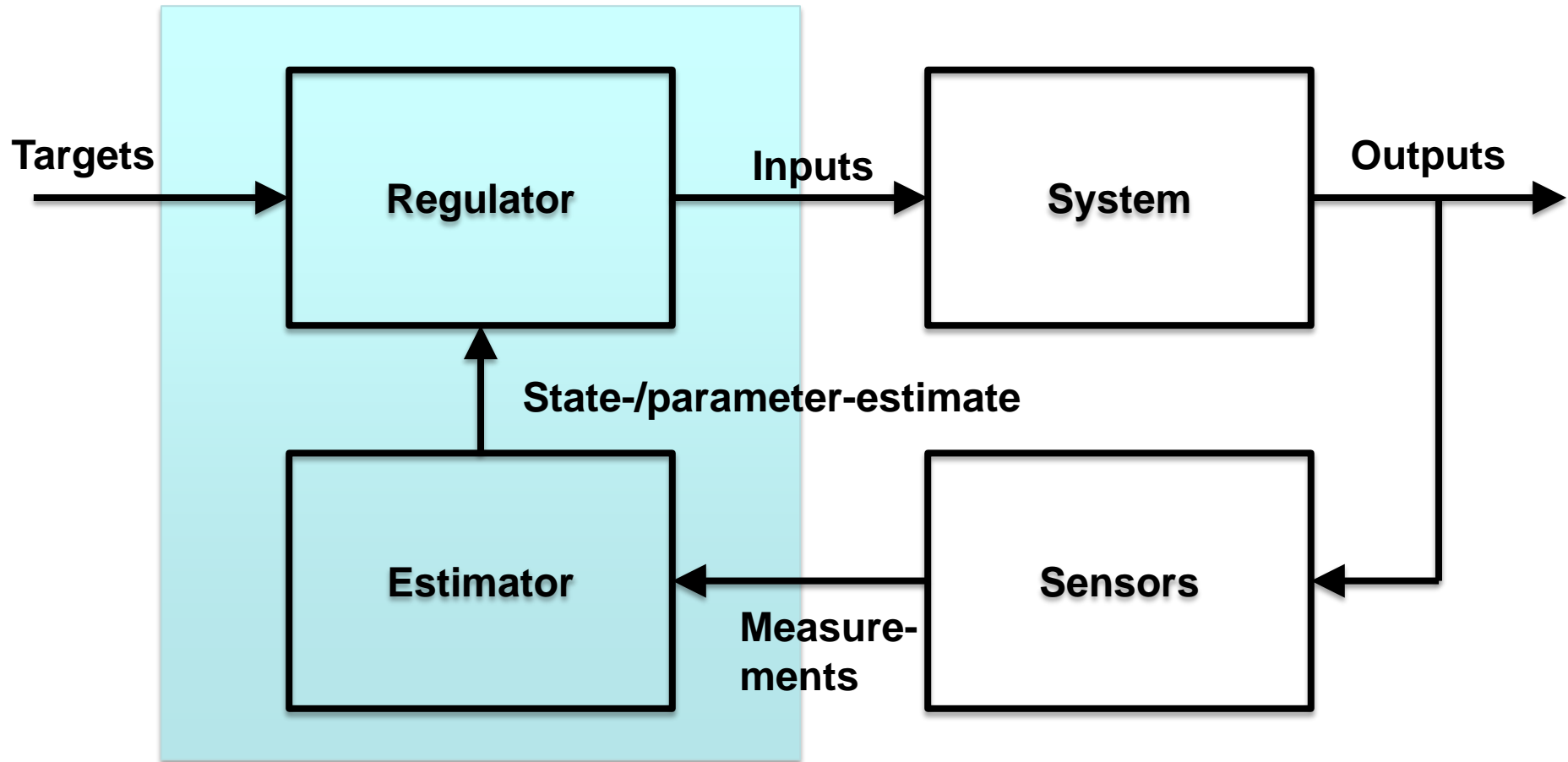
MODEL PREDICTIVE CONTROL

Systematic Model Building

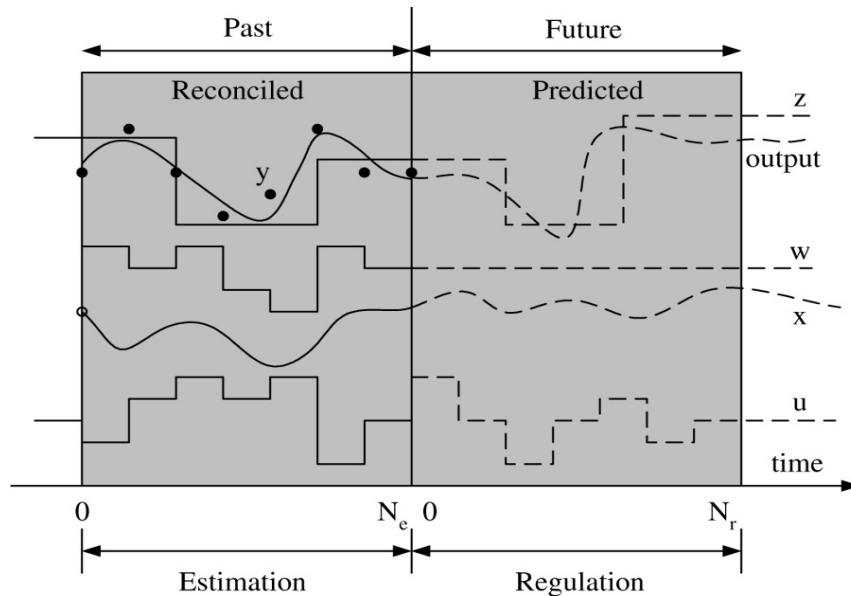


Model Predictive Controller

Model Predictive Controller



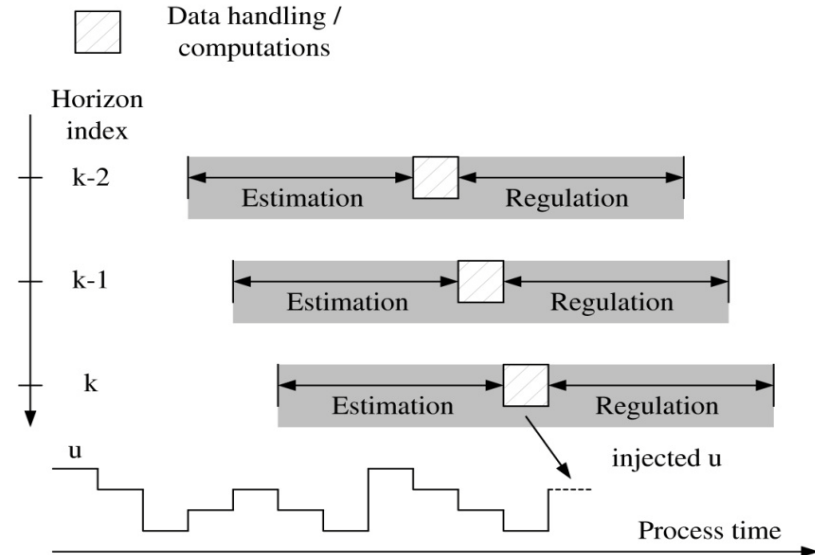
Model Predictive Control



MPC

=

Estimation + Regulation



**Moving horizon
implementation**

Filtering and Prediction - EKF

$$d\mathbf{x}(t) = f(\mathbf{x}(t), u(t), \theta)dt + \sigma d\omega(t)$$

$$\mathbf{y}(t_k) = g(\mathbf{x}(t_k), \theta) + \mathbf{v}(t_k)$$

Filtering

$$e_k = y_k - g(\hat{x}_{k|k-1}, \theta)$$

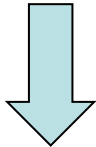
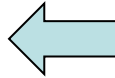
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k e_k$$

$$P_{k|k} = P_{k|k-1} - K_k R_{k|k-1} K'_k$$

$$C_k = \frac{\partial g}{\partial x}(\hat{x}_{k|k-1}, \theta)$$

$$R_{k|k-1} = C_k P_{k|k-1} C'_k + R_k$$

$$K_k = P_{k|k-1} C'_k R_{k|k-1}^{-1}$$



Prediction

$$\frac{d\hat{x}_k(t)}{dt} = f(\hat{x}_k(t), u(t), \theta)$$

$$\hat{x}(t_k) = \hat{x}_{k|k}$$

$$\frac{dP_k(t)}{dt} = A(t)P_k(t) + P_k(t)A(t)' + \sigma\sigma'$$

$$P_k(t_k) = P_{k|k}$$

$$A(t) = \frac{\partial f}{\partial x}(\hat{x}_k(t), \theta)$$



$$\hat{x}_{k+1|k} = \hat{x}_k(t_{k+1})$$

$$P_{k+1|k} = P_k(t_{k+1})$$

Optimal Control Problem

$$\begin{aligned} \min \quad & \phi = \int_{t_k}^{t_k+T} g(x(t), u(t)) dt \\ \text{s.t.} \quad & x(t_k) = \hat{x}_{k|k} \\ & \dot{x}(t) = f(x(t), u(t)) \quad t \in [t_k, t_k + T] \\ & c(x(t), u(t)) \geq 0 \quad t \in [t_k, t_k + T] \end{aligned}$$

Integrator (Runge-Kutta Methods)

- DOPRI54 (non-stiff systems)
- ESDIRK12 / ESDIRK23 / ESDIRK34 (stiff systems)

Sensitivities

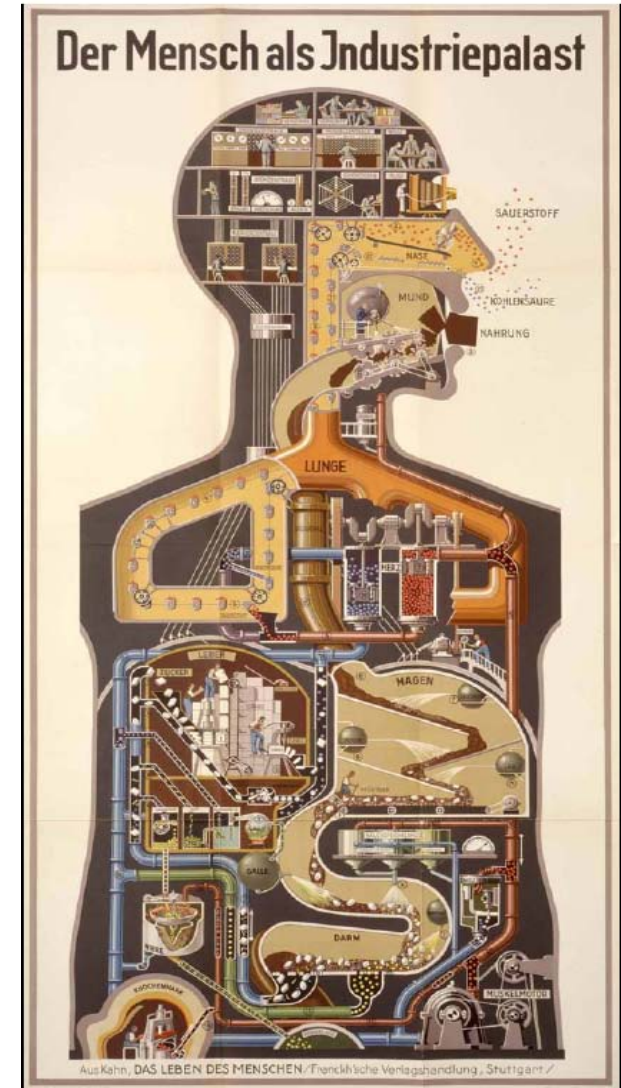
- Forward, Adjoint

Optimization

- SQP
- Single-shooting
- Multiple-shooting

Insulin Administration Systems for People with Diabetes

The Artificial Pancreas

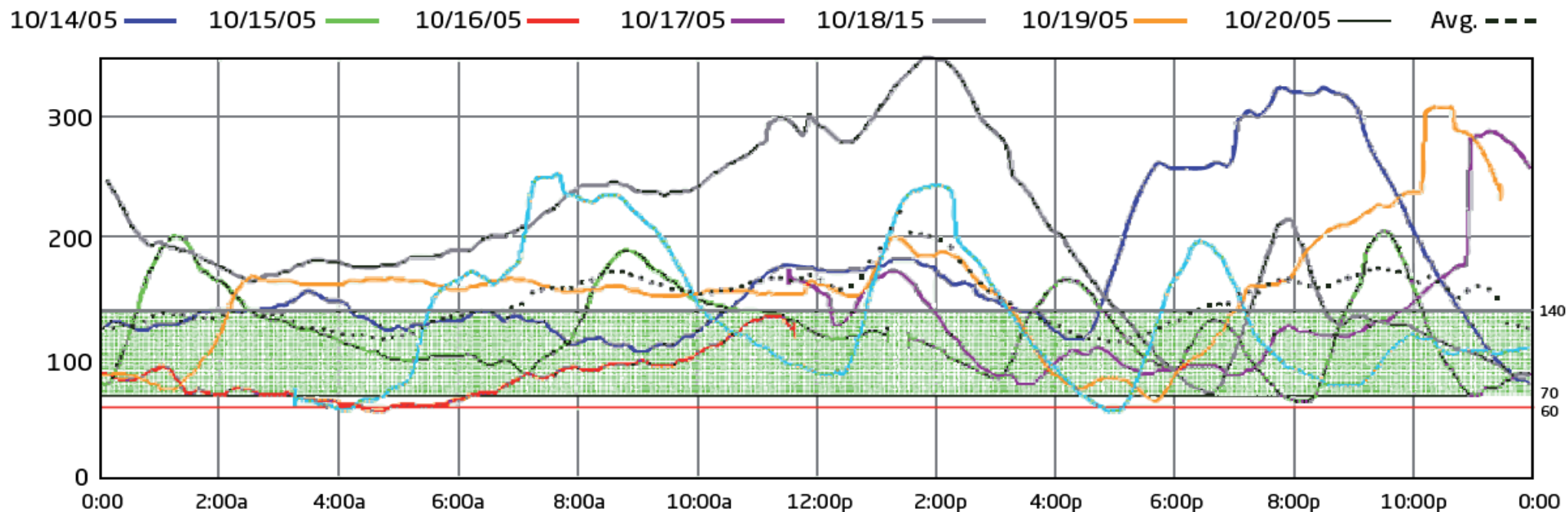


Blood glucose control

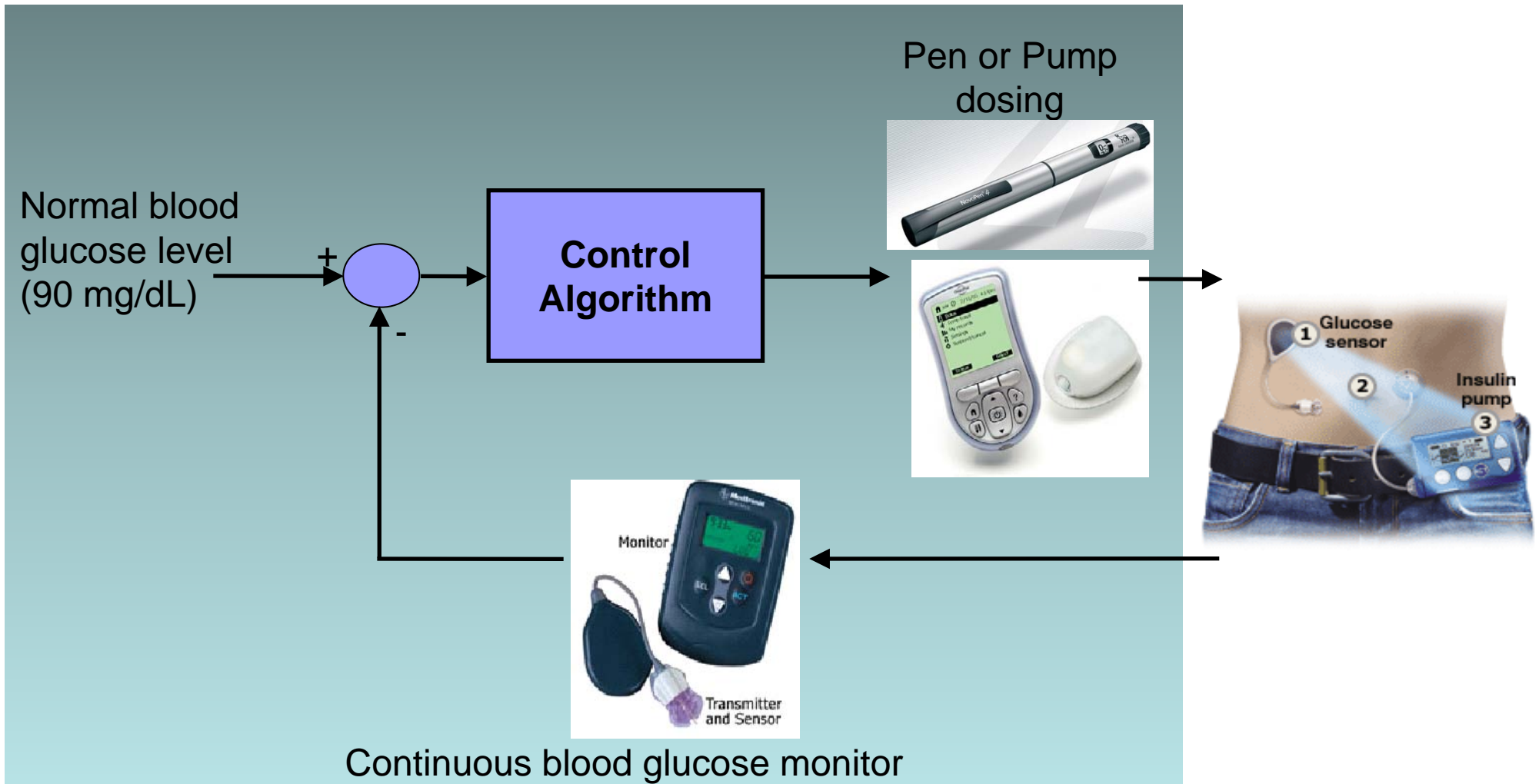
The blood glucose must stay within certain upper and lower bounds!

- Too low: coma (immediate effect)
- Too high: blindness and other long-term effects

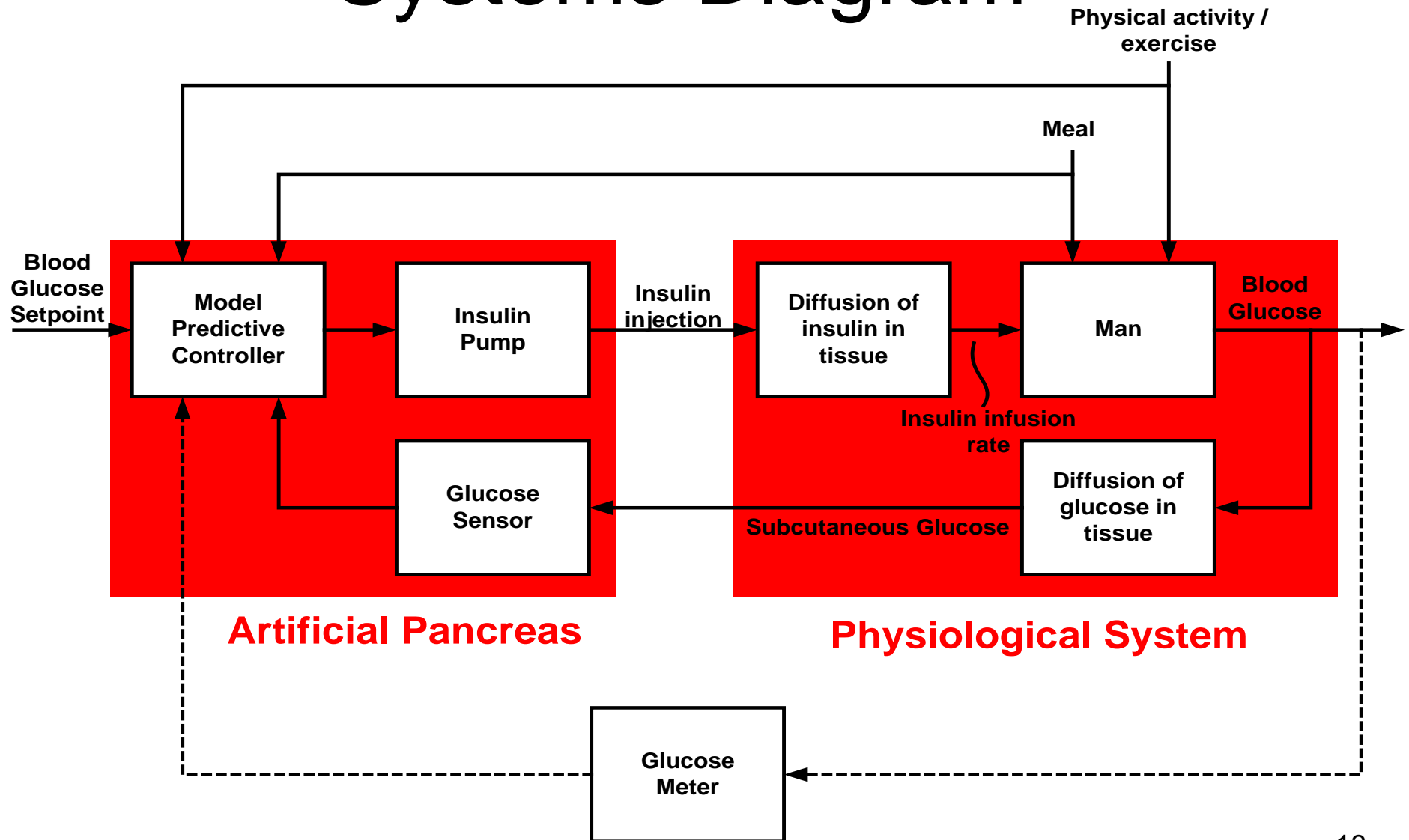
Sensor Data (mg/dL)



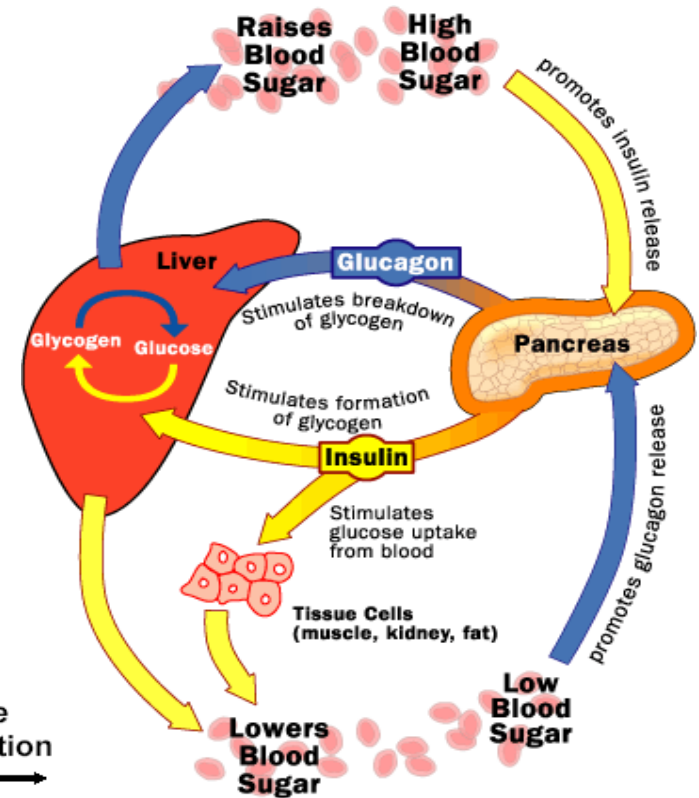
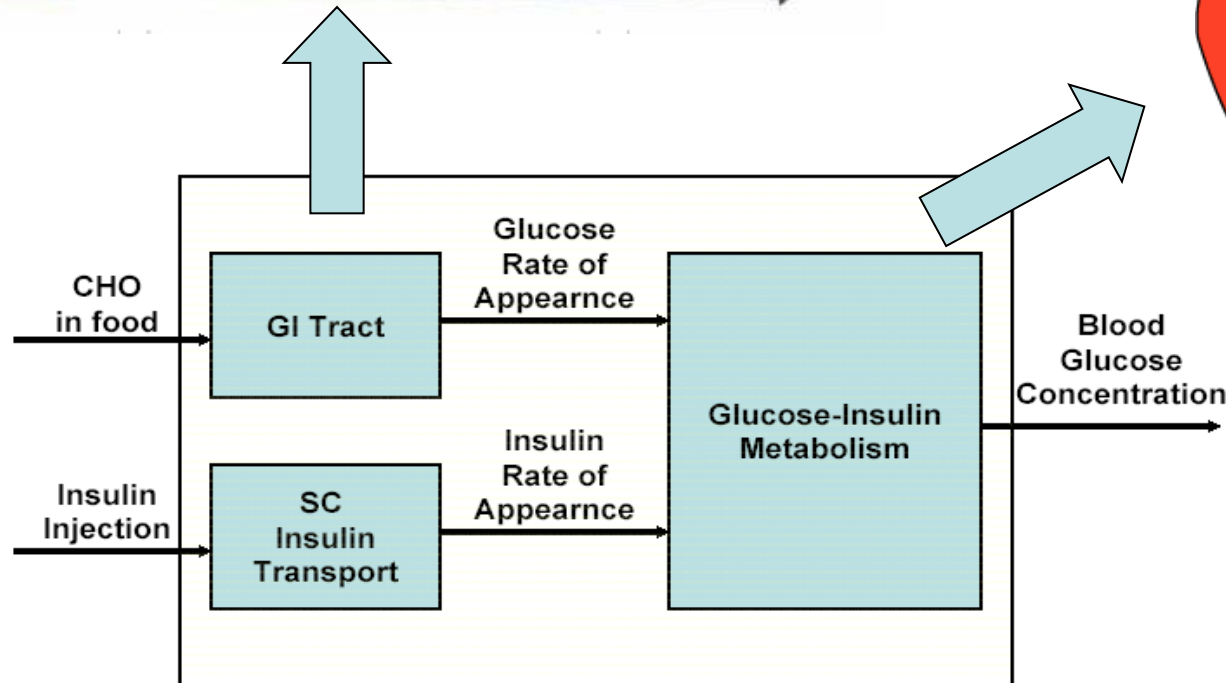
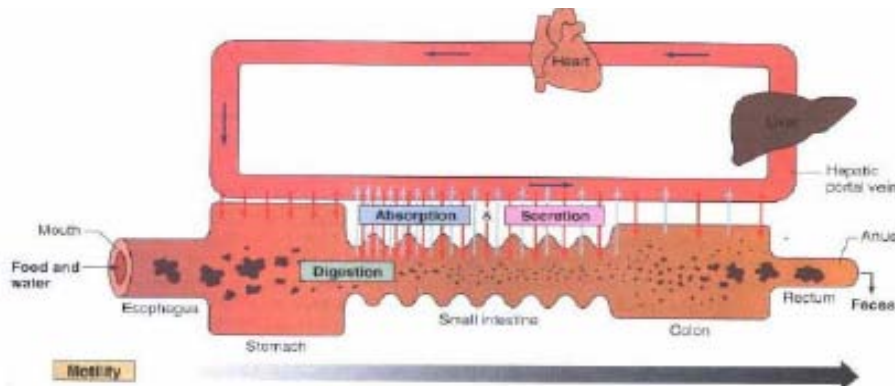
Insulin Administration Systems



Systems Diagram

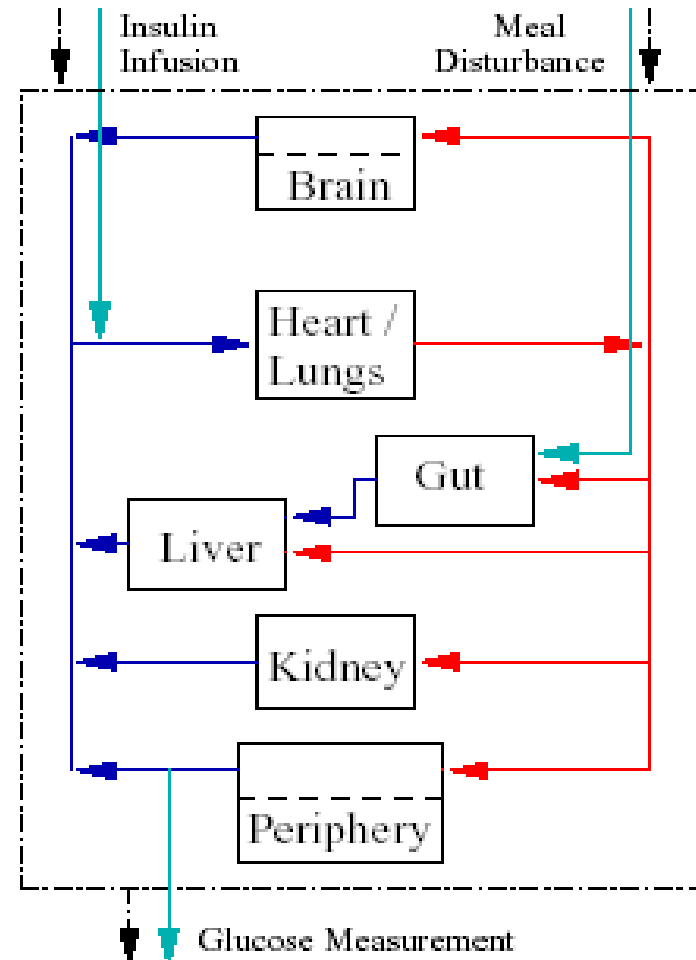
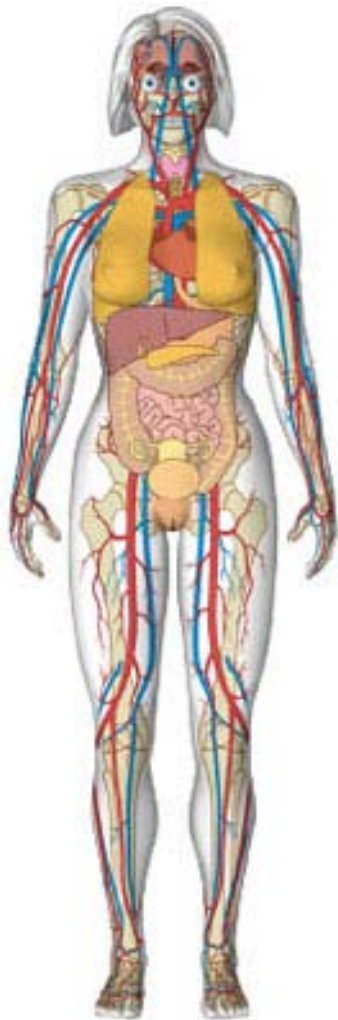


Physiological Models – The Virtual Patient

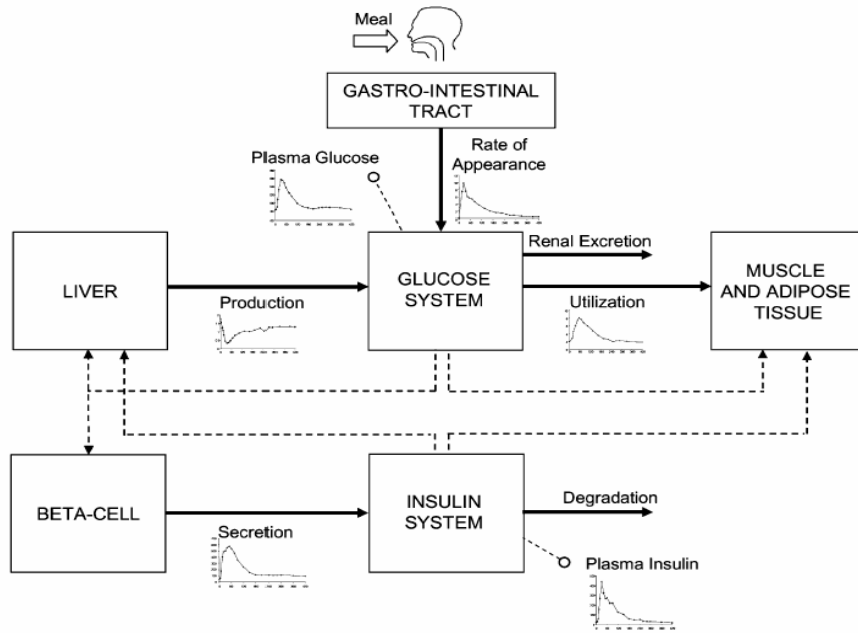
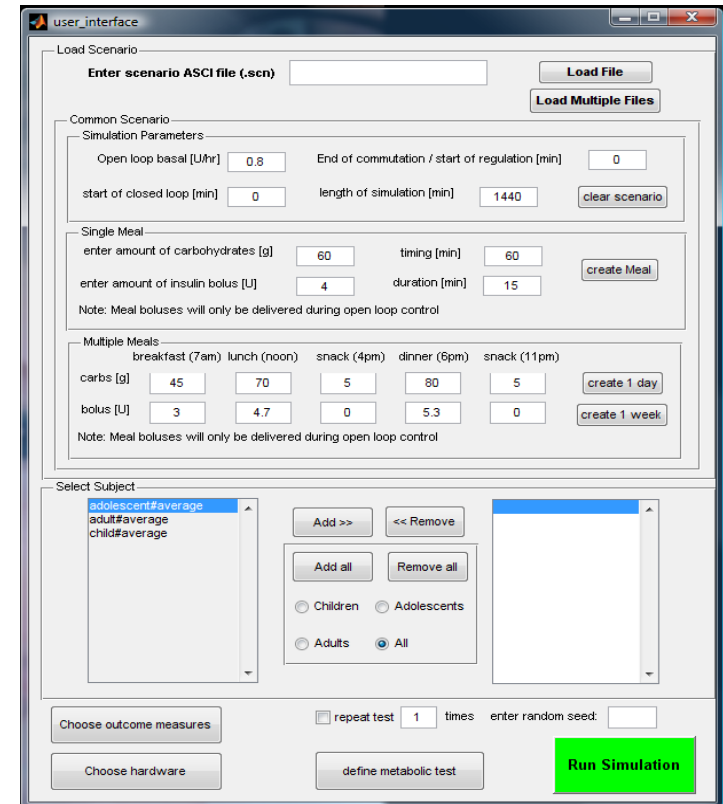


Physiological Models

– A Compartment Model



The Cobelli Model – and the Virtual Clinic

The **user_interface** window contains the following sections:

- Load Scenario**: Includes fields for **Enter scenario ASCII file (.scn)** and buttons for **Load File** and **Load Multiple Files**.
- Common Scenario**:
 - Simulation Parameters**: Fields for **Open loop basal [U/hr]** (0.8), **End of commutation / start of regulation [min]** (0), **start of closed loop [min]** (0), and **length of simulation [min]** (1440). Includes a **clear scenario** button.
 - Single Meal**: Fields for **enter amount of carbohydrates [g]** (60), **timing [min]** (60), **enter amount of insulin bolus [U]** (4), and **duration [min]** (15). Includes a **create Meal** button.
 - Multiple Meals**: A table for defining meals over a day/week.

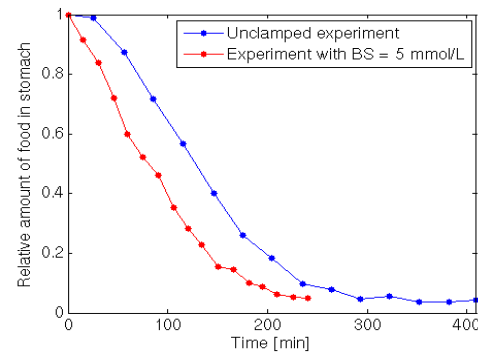
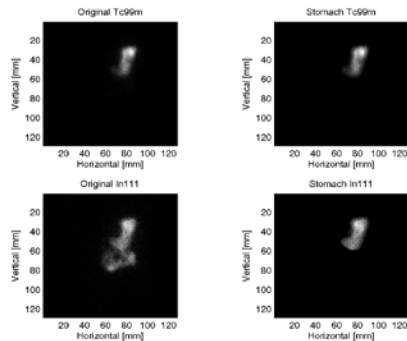
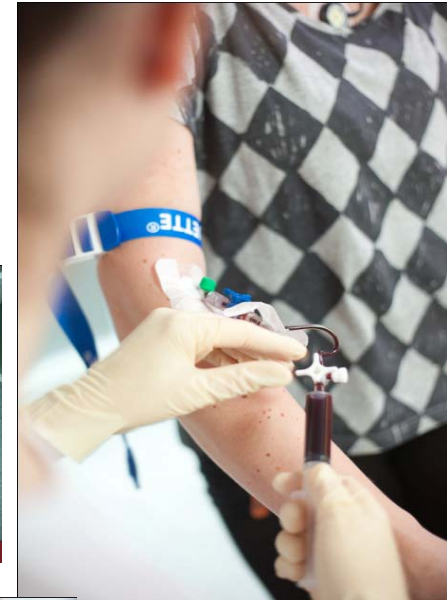
	breakfast (7am)	lunch (noon)	snack (4pm)	dinner (6pm)	snack (11pm)
carbs [g]	45	70	5	80	5
bolus [U]	3	4.7	0	5.3	0

Includes **create 1 day** and **create 1 week** buttons.
- Select Subject**: A list box with subjects like **adolescent#average**, **adult#average**, and **child#average**. Includes **Add >>**, **<< Remove**, **Add all**, and **Remove all** buttons. Radio buttons for **Children**, **Adolescents**, **Adults**, and **All** (selected).
- Controls**: **Choose outcome measures**, **Choose hardware**, **define metabolic test**, **repeat test** (checkbox), **1 times**, **enter random seed:** (field), and a large green **Run Simulation** button.

- Developed by
 - University of Virginia
 - University of Padova
 - Juvenile Diabetes Research Foundation
- 30 virtual patients in the version that we have a license to
- The full version including 300 patients approved to substitute animal tests by FDA

Clinical Trials

- Gastric emptying trials (left)
- Clinical trials collecting data for modeling (right)



Example of a Clinical Experiment



8:00
Insert
lines

10:00
Meal/
bolus

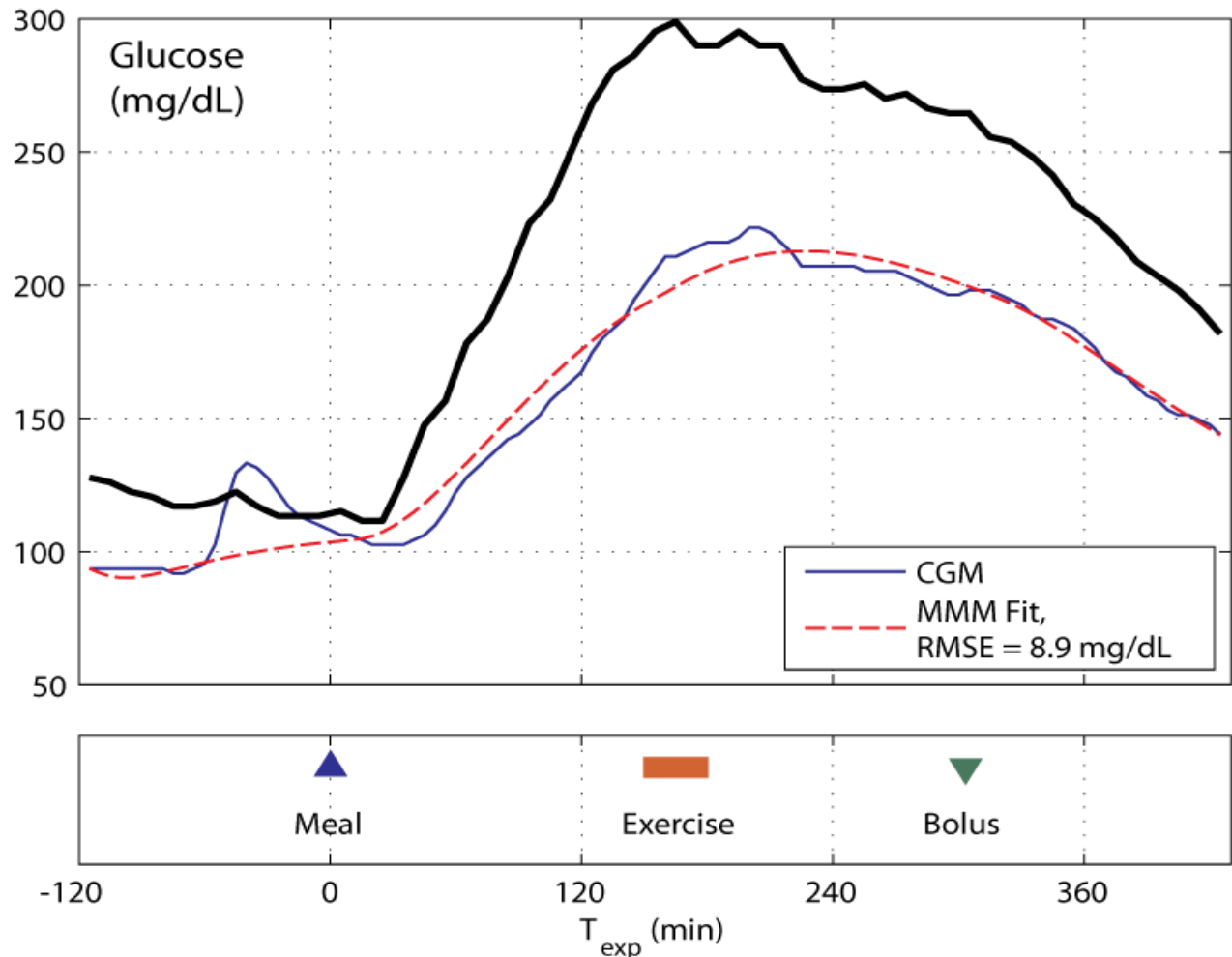
12:30
Exercise

15:00
Snack

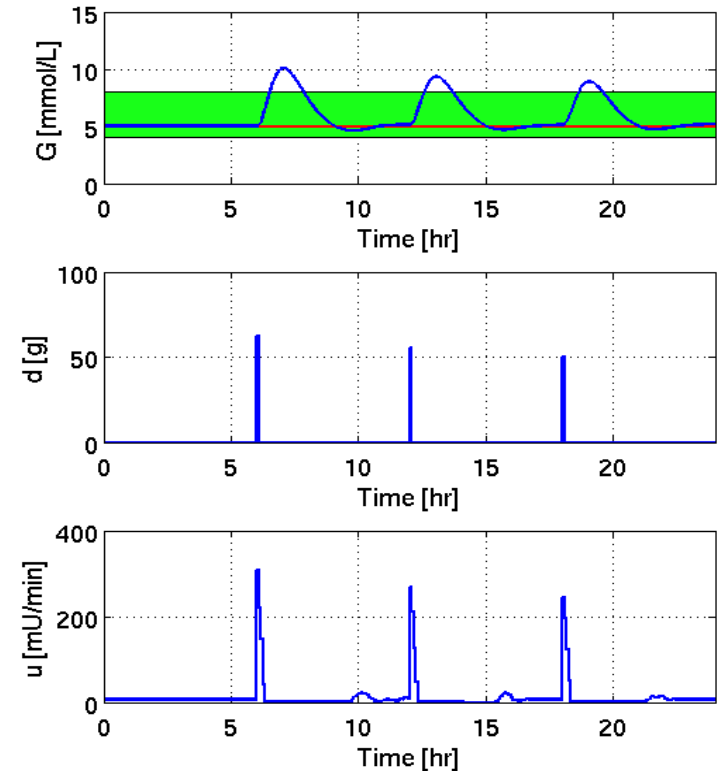
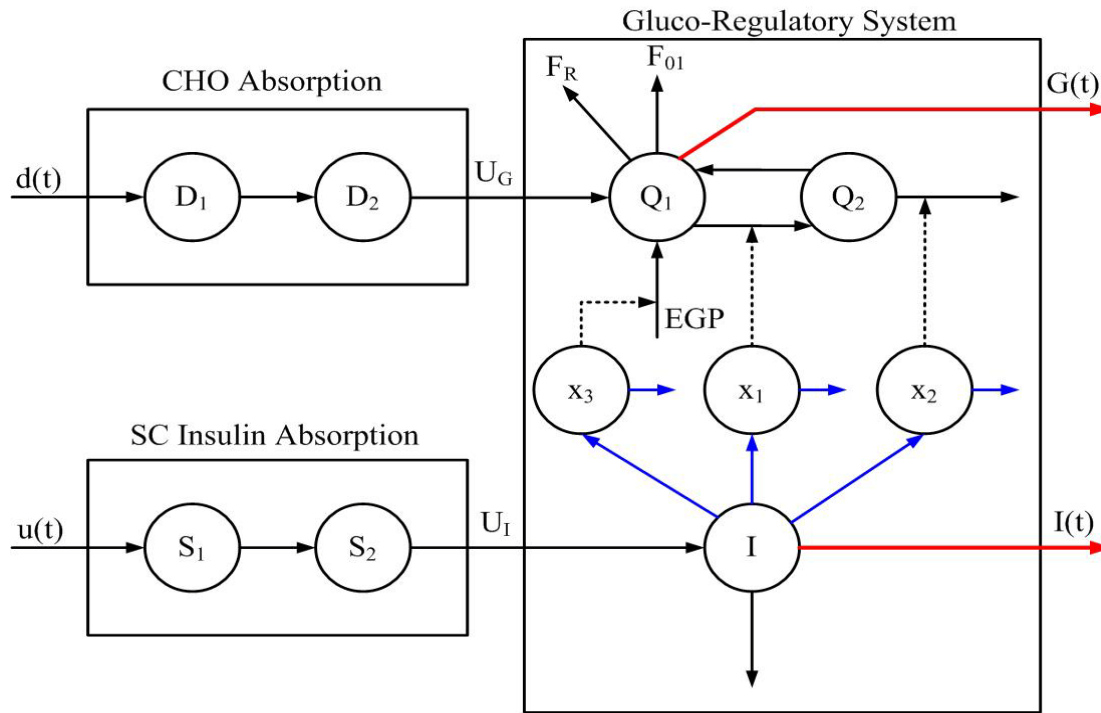


Identification Results

- Representative model fit
- Very accurate model fit
- **Note:**
Model is fit to the CGM signal. The intravenous glucose concentration is also shown (thick line) for comparison



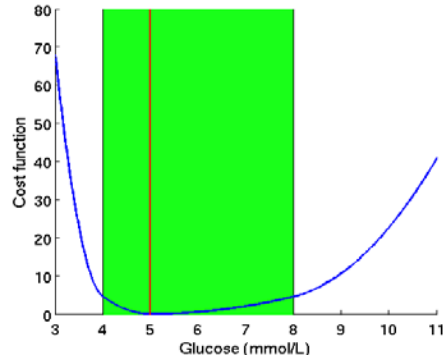
The DTU Type 1 Diabetes Model



$$d\mathbf{x}(t) = f(\mathbf{x}(t), u(t), d(t))dt + \sigma d\boldsymbol{\omega}(t)$$

$$\mathbf{y}(t_k) = g(\mathbf{x}(t_k)) + \mathbf{v}(t_k)$$

Insulin Administration Strategies



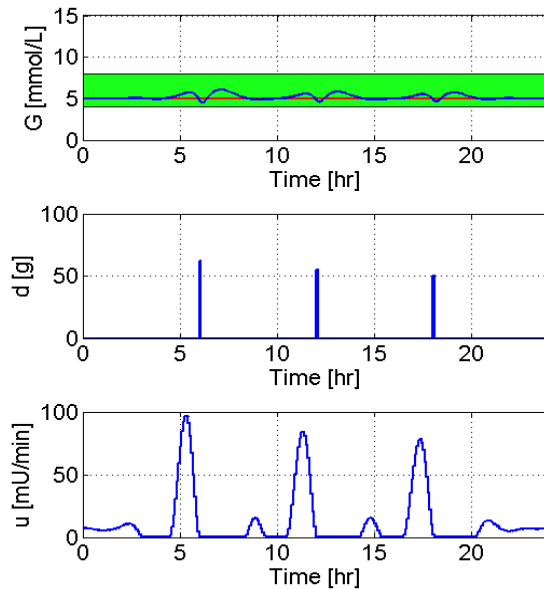
$$\min \quad \phi = \int_{t_k}^{t_k+T} g(x(t), u(t)) dt$$

$$s.t. \quad x(t_k) = \hat{x}_{k|k}$$

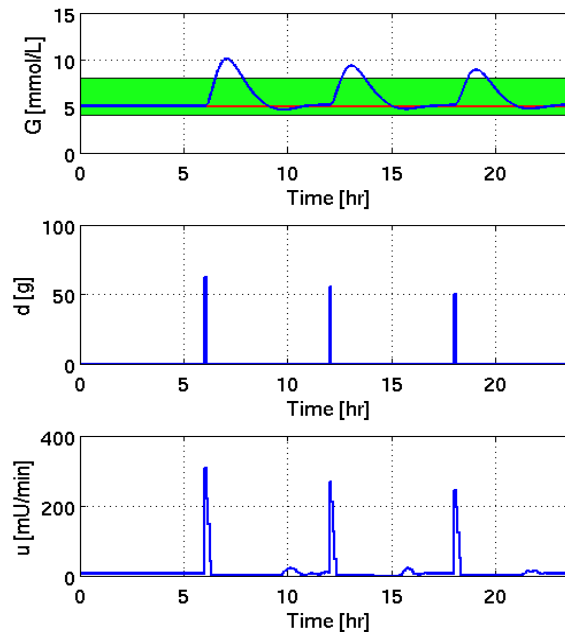
$$\dot{x}(t) = f(x(t), u(t), d(t)) \quad t \in [t_k, t_k + T]$$

$$c(x(t), u(t)) \geq 0 \quad t \in [t_k, t_k + T]$$

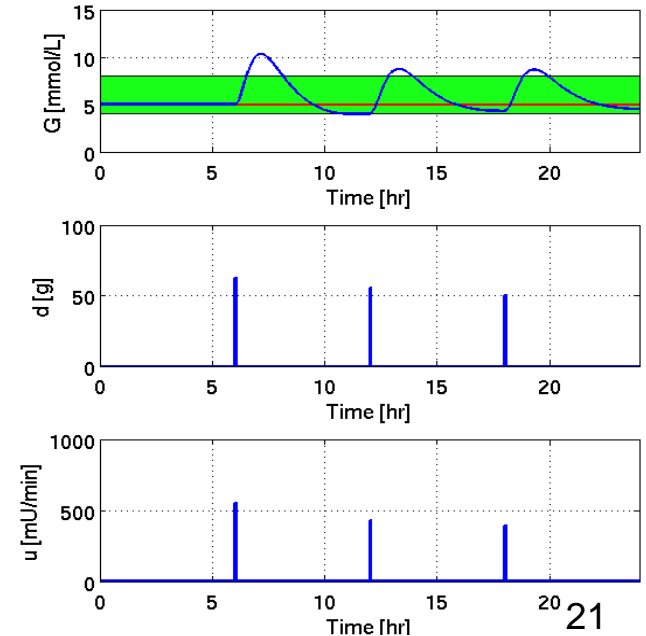
NMPC Pre-meal Insulin Allowed



NMPC No Pre-meal Insulin

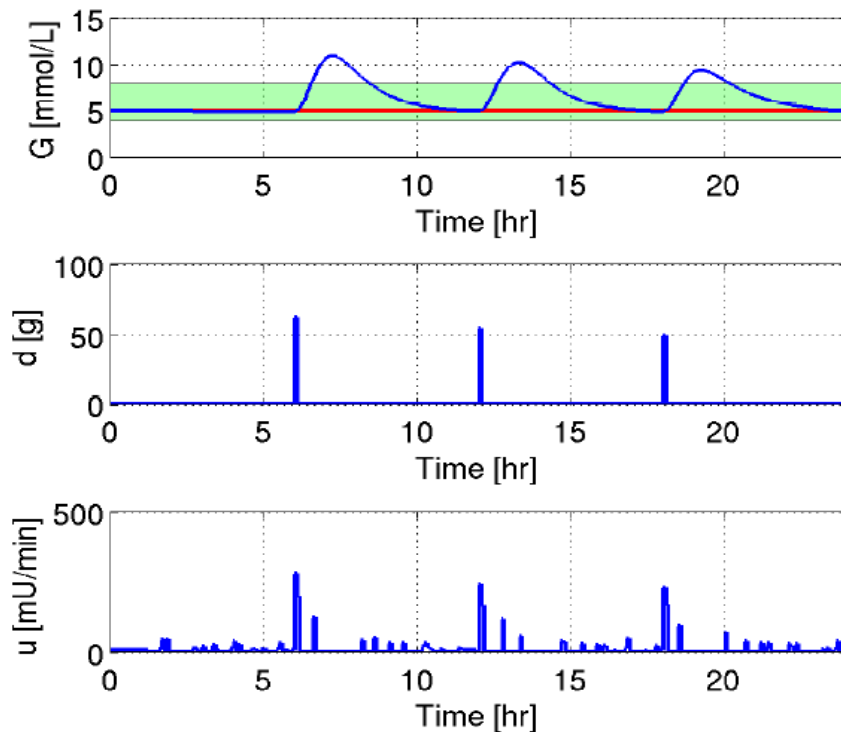


MDI (Pen Based) Insulin Treatment

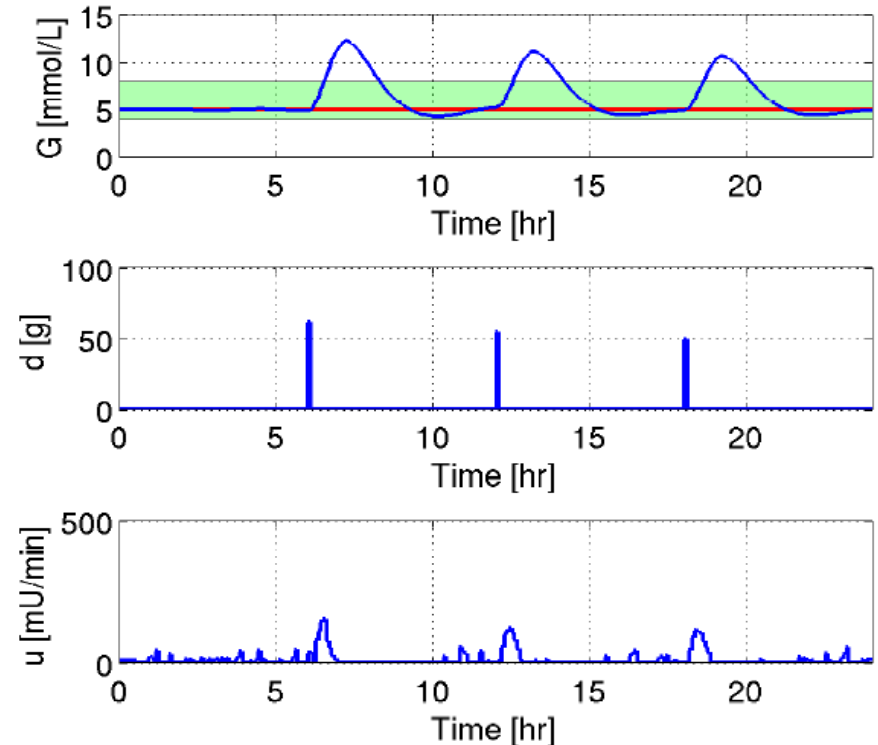


Closed-Loop Studies by NMPC

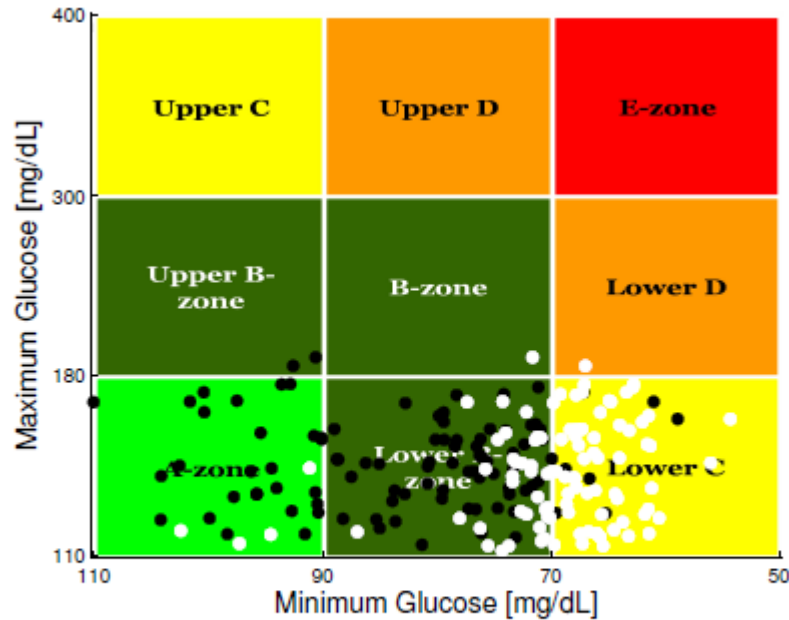
Meals announced at meal-time



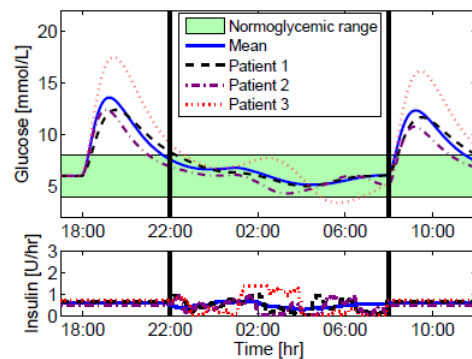
Meals not announced



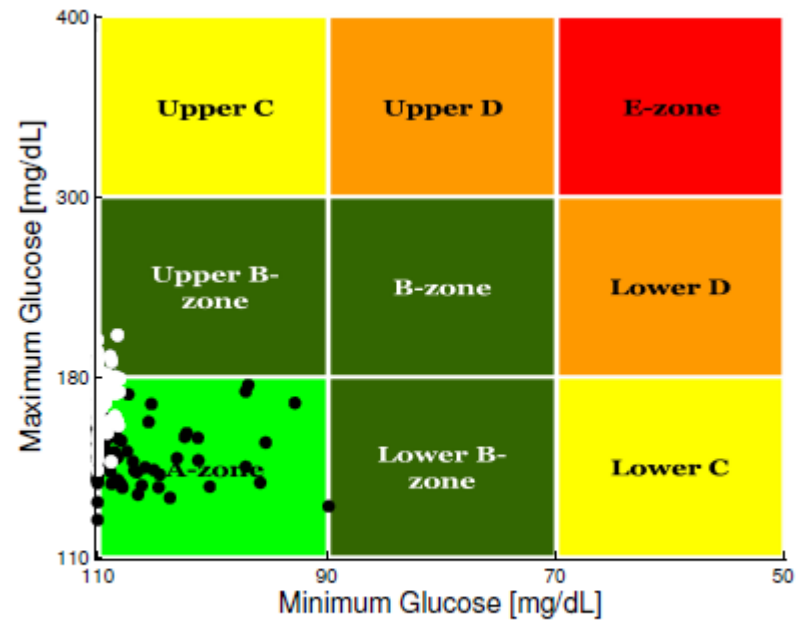
Overnight Stabilization – A Cohort



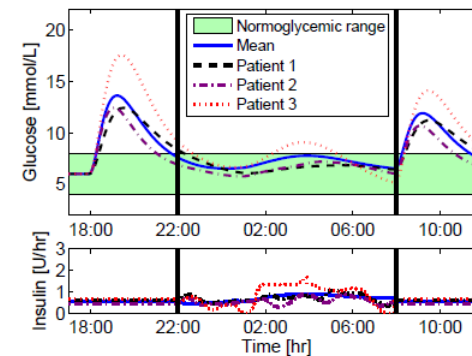
(a) Insulin sensitivity increases by 30%



(a) Insulin sensitivity increases by 30%

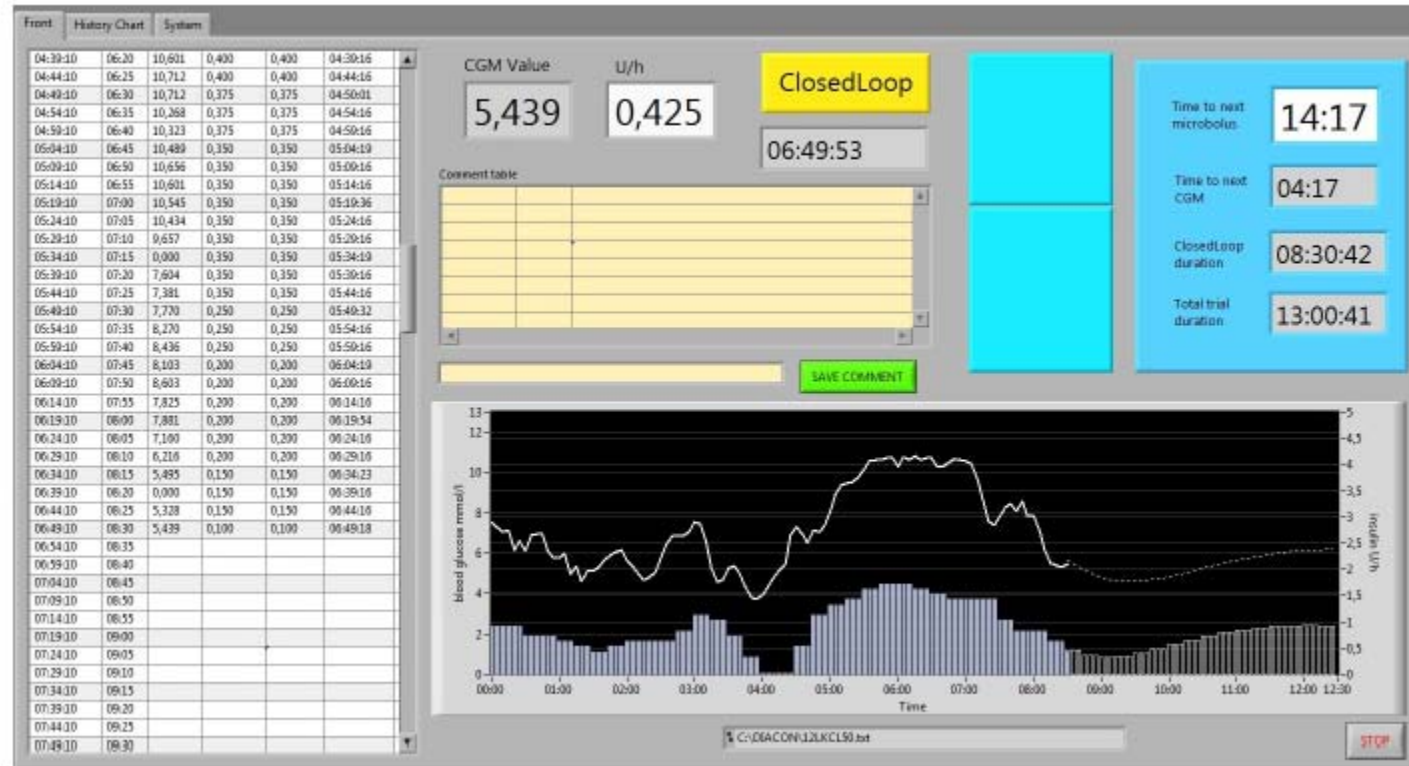


(b) Insulin sensitivity decreases by 30%



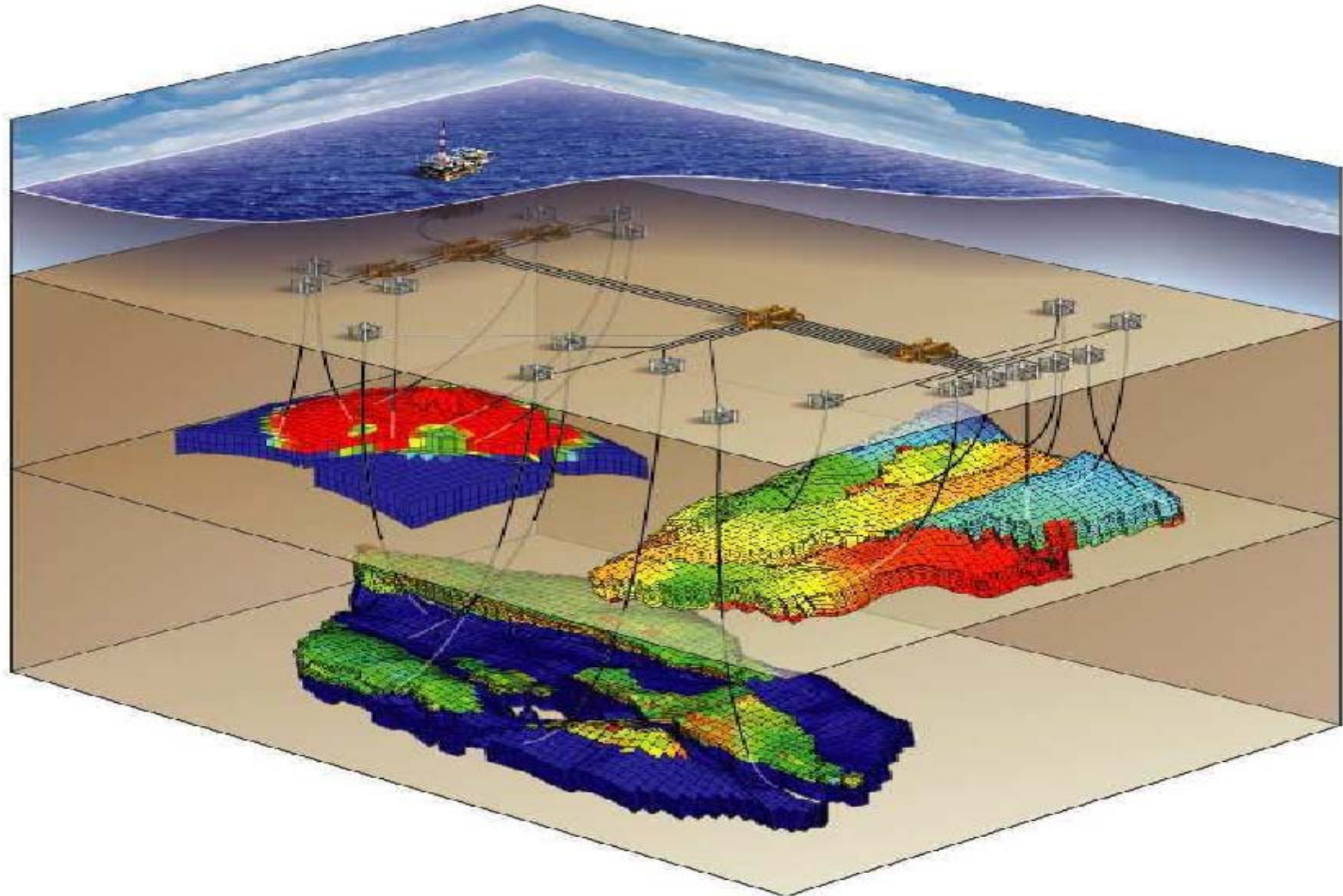
(b) Insulin sensitivity decreases by 30%

Clinical Closed Loop Studies

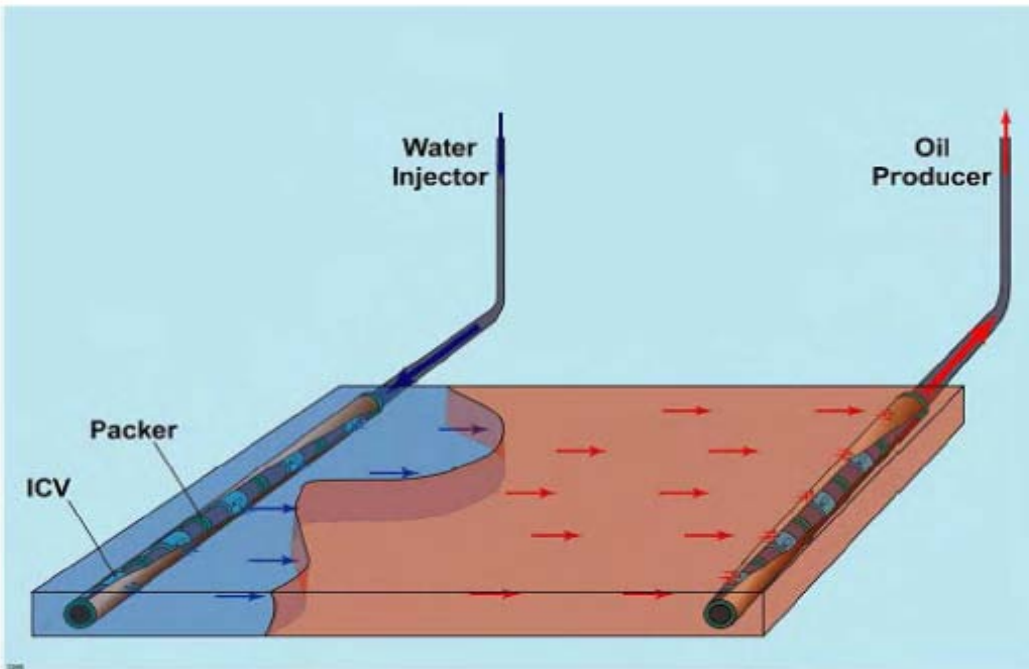


MODEL PREDICTIVE CONTROL FOR MANAGEMENT OF OIL RESERVOIRS

An Offshore Oil Reservoir



Mathematical Model



The mass conservation of water and oil

$$\frac{\partial}{\partial t} C_w(P_w, S_w) = -\nabla \cdot \mathbf{F}_w(P_w, S_w) + Q_w$$

$$\frac{\partial}{\partial t} C_o(P_o, S_o) = -\nabla \cdot \mathbf{F}_o(P_o, S_o) + Q_o$$

The mass concentrations

$$C_w = \phi \rho_w(P_w) S_w$$

$$C_o = \phi \rho_o(P_o) S_o$$

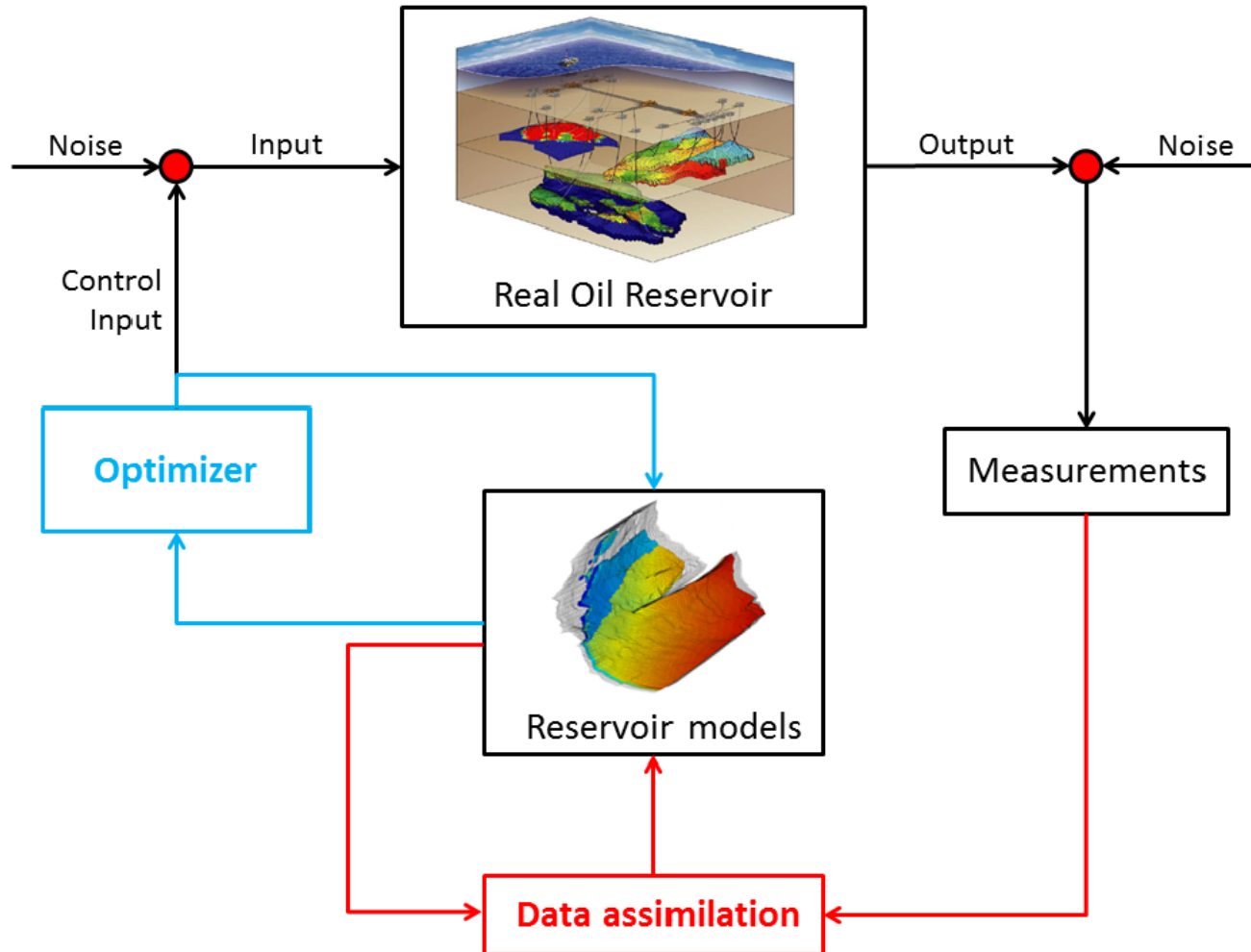
Fluxes through the porous medium

$$\mathbf{F}_w = \rho_w(P_w) \mathbf{u}_w(P_w, S_w)$$

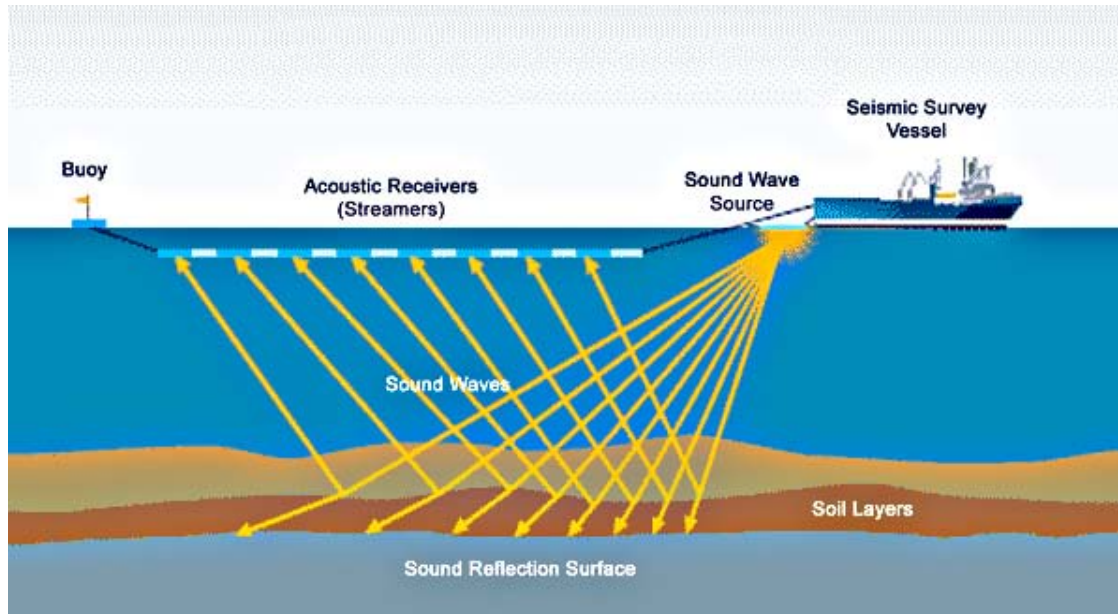
$$\mathbf{F}_o = \rho_o(P_o) \mathbf{u}_o(P_o, S_o)$$

$$\frac{d}{dt} g(x(t)) = f(x(t), u(t))$$

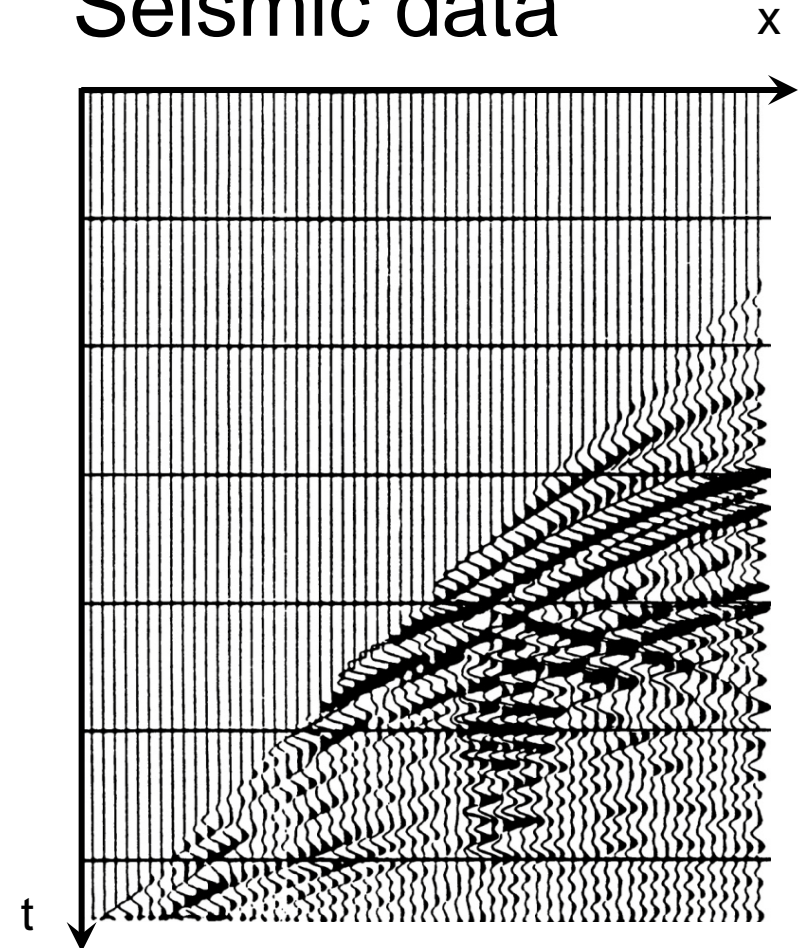
Closed-Loop Reservoir Management



Experimental setup

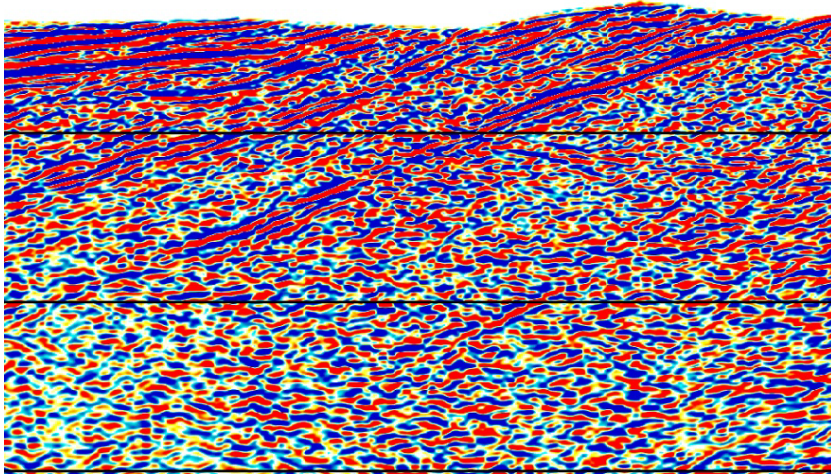


Seismic data

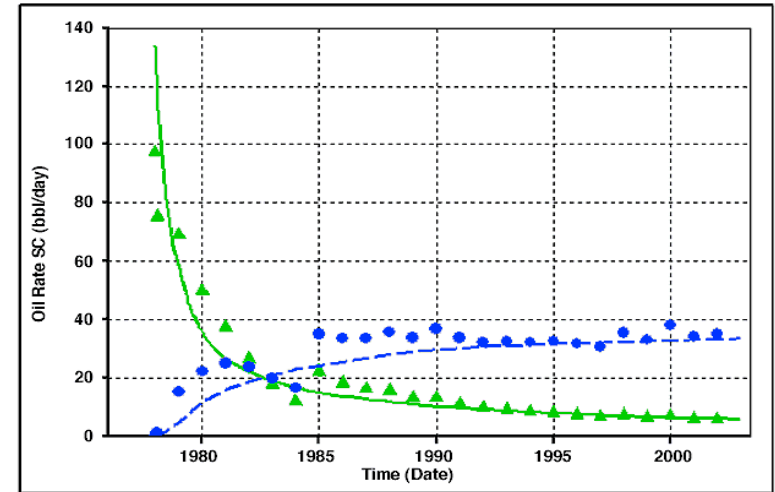


The Computational Challenge: Integrated Inversion of Seismic and Production Data

Seismic data

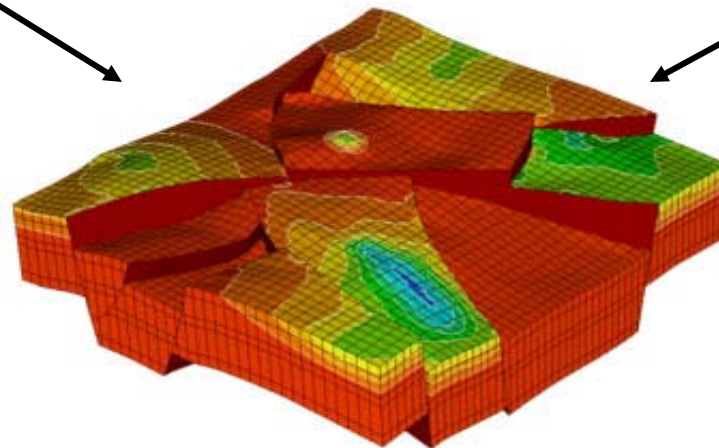


Oil production data



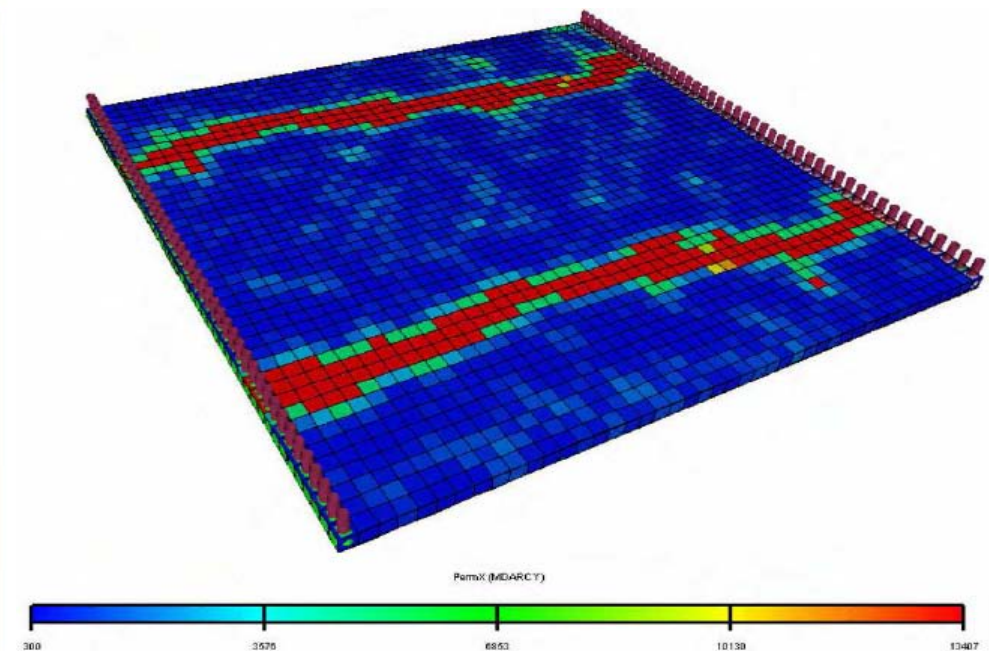
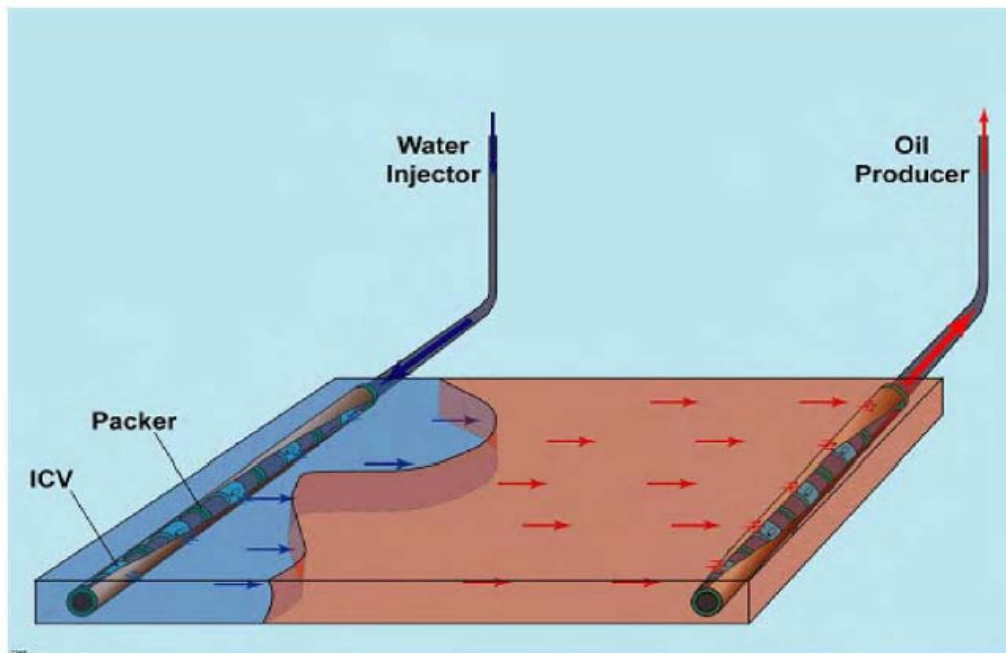
Seismic inversion

History matching



History Matching (State and Parameter Est) + Seismic Data

=> Permeability Field



Optimization Problem

$$\begin{aligned} \min_{[x(t), u(t)]_{t_0}^{t_f}} \quad & \int_{t_0}^{t_f} J(x(t), u(t)) dt \\ \text{s.t.} \quad & \frac{d}{dt} g(x(t)) = f(x(t), u(t)) \\ & x(t_0) = x_0 \\ & u_{min} \leq u(t) \leq u_{max} \\ & -u_{min}^{\Delta} \leq \frac{d}{dt} u(t) \leq u_{max}^{\Delta} \end{aligned}$$

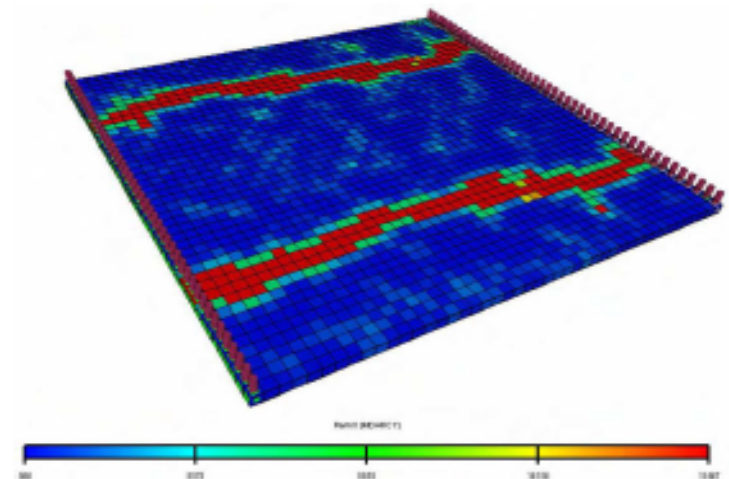
- Single-Shooting and SQP
- ESDIRK methods tailored for this problem
- Gradients computed by the adjoint method

Objective Function is the Net Present Value

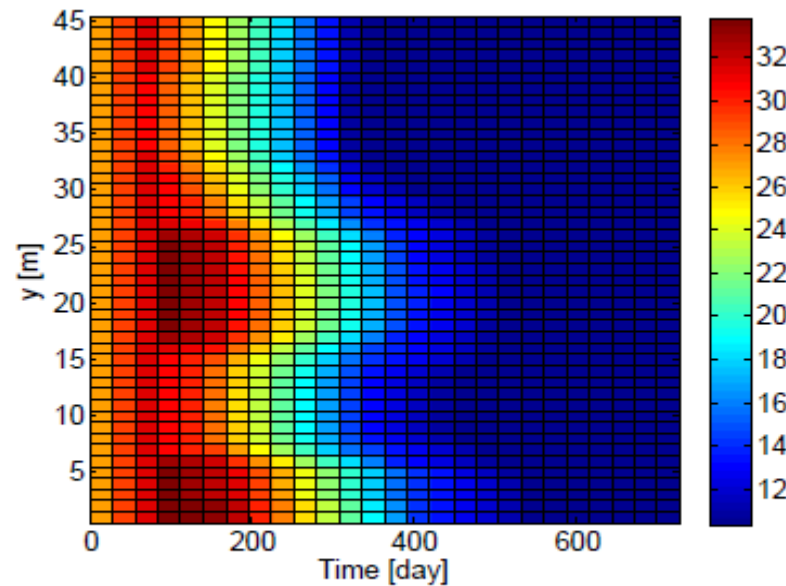
$$NPV = \sum_{k=0}^{N-1} \left[\frac{h_k}{(1-d)^{t_k/\tau}} \left(\sum_{j=1}^{N_{pro}} [r_{op} Q_{k,j}^{op} - r_{wp} Q_{k,j}^{wp}] - \sum_{j=1}^{N_{inj}} r_{wi} Q_{k,j}^{wi} \right) \right]$$

Adjust the injectors and producers by taking the oil price, the water injection cost and the water separation cost into account.

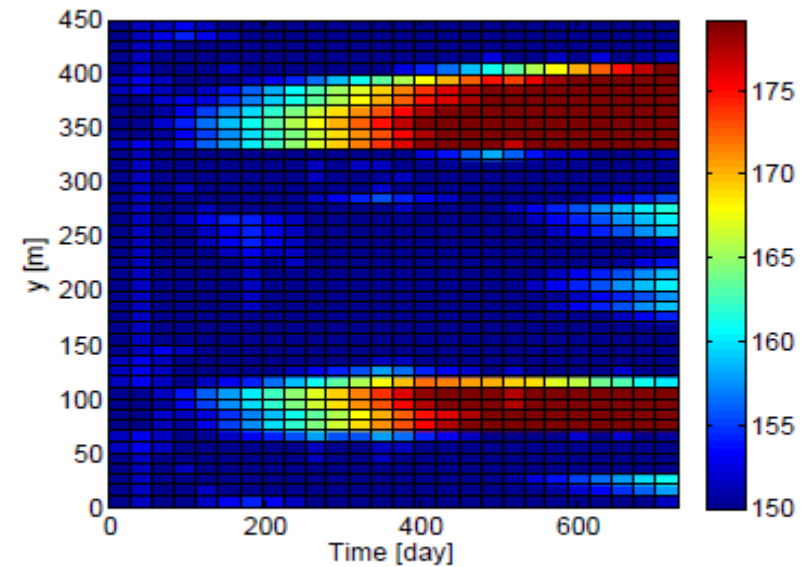
- Rate constrained injector on the left.
- BHP constrained producer to the right.
- 2×45 individually controllable sections.
- Max. 2 PV's injected over two years.
- Injection rates and BHP's are updated once a month.



Optimal Oil Field Development - MVs

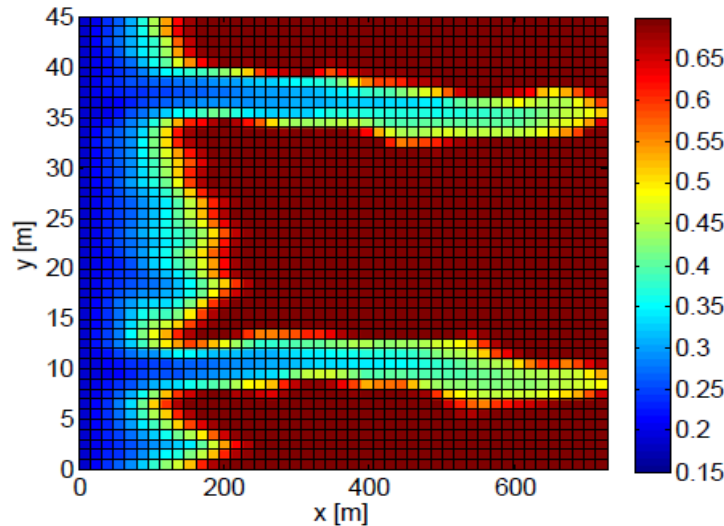


(a) Individual injection rates (m³/day) of 45 injectors.

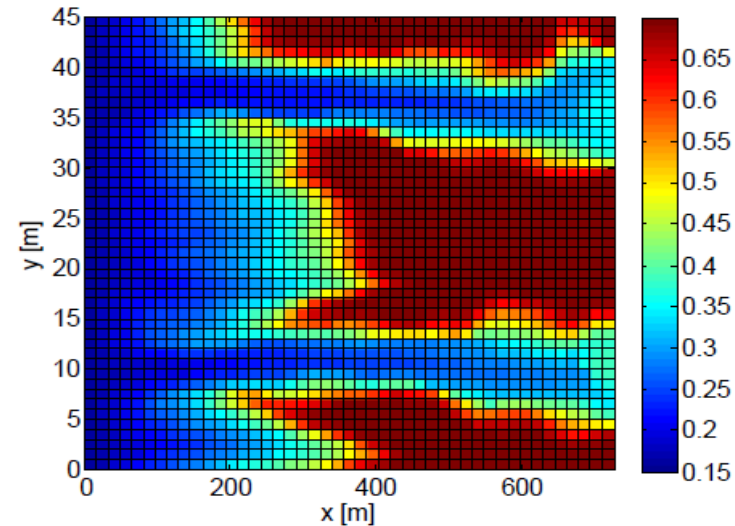


(b) Individual BHP's (atm) of 45 producers.

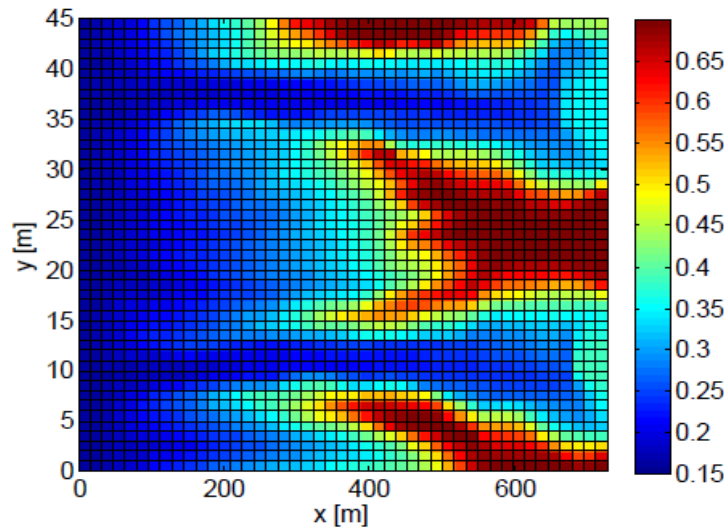
Optimal Oil Field Development - CVs



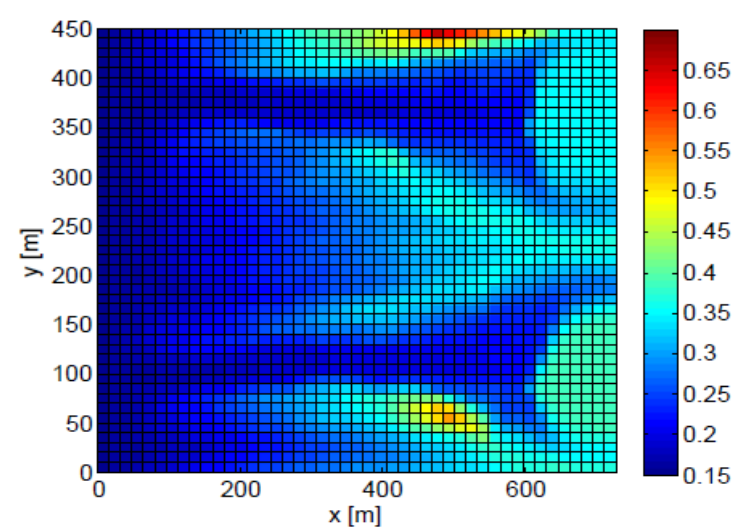
(a) 0.14 PV injected after 50 days.



(b) 0.41 PV injected after 125 days.



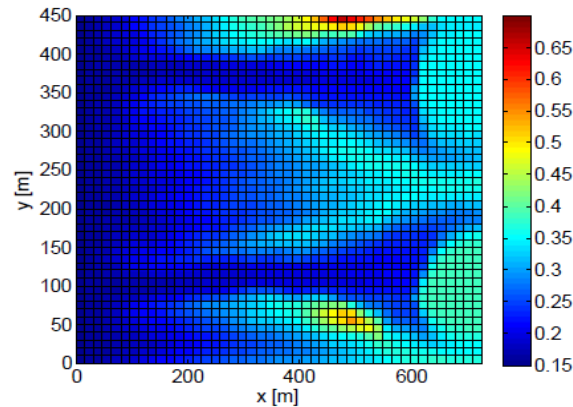
(c) 0.64 PV injected after 200 days.



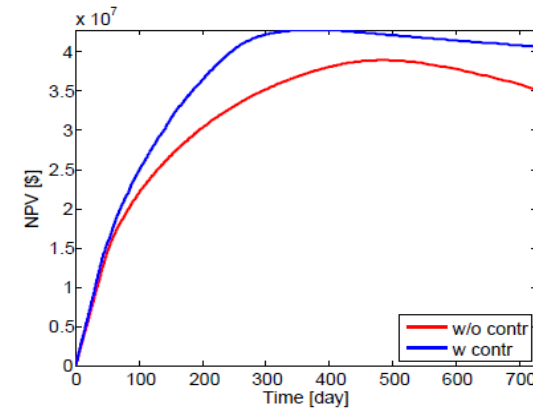
(d) 1.00 PV injected after 374 days.

Oil Field Development Using NMPC

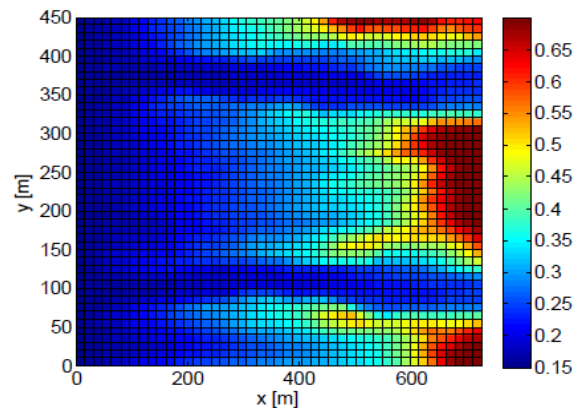
With control strategy, 374 days:



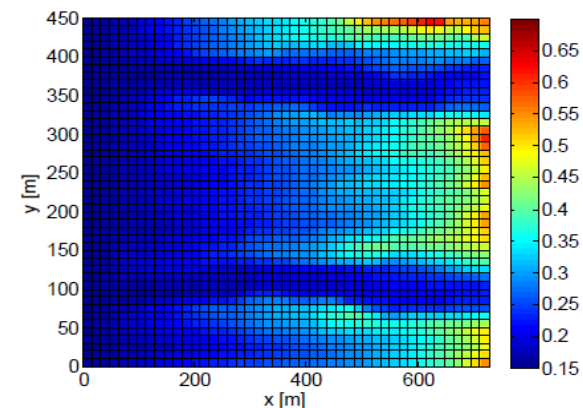
Net present value over 2 years:



No control strategy, 374 days:

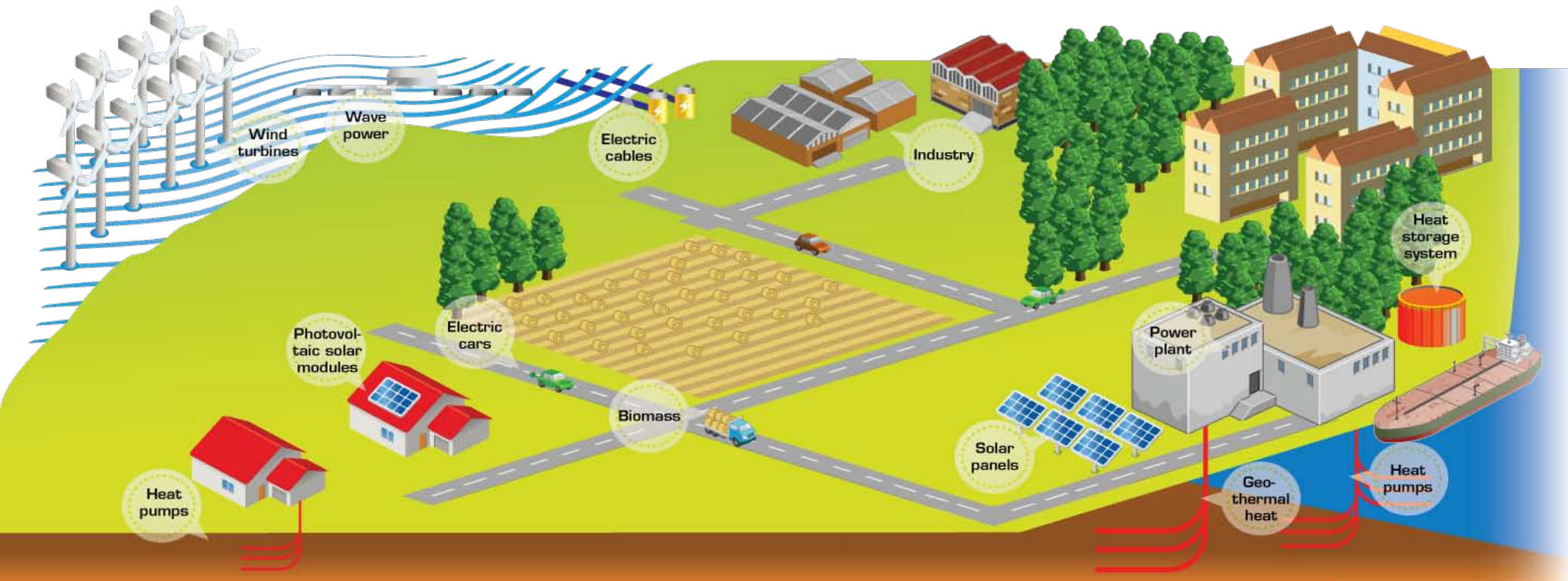


No control strategy, 484 days:

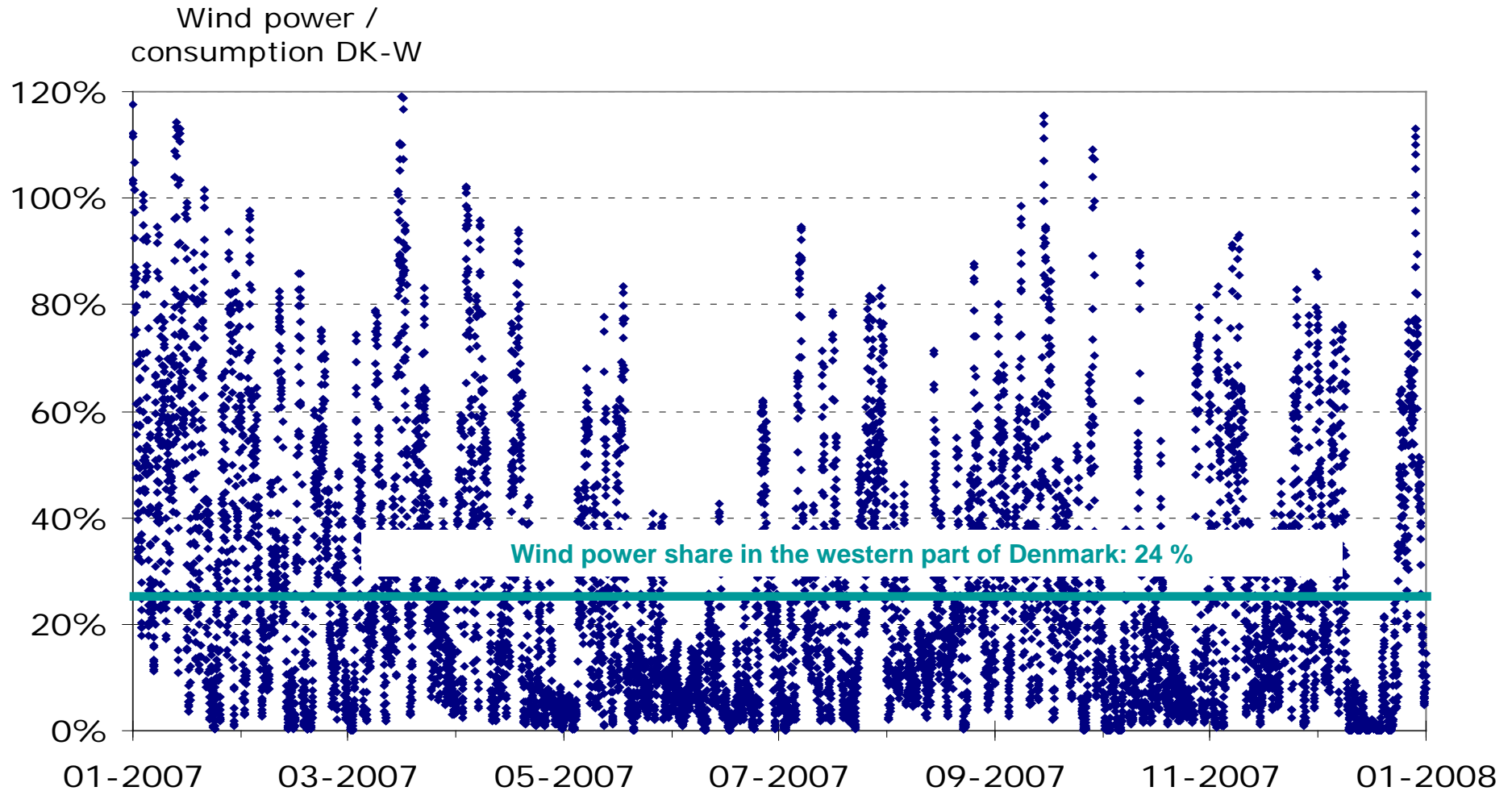


MODEL PREDICTIVE CONTROL OF SMART ENERGY SYSTEMS

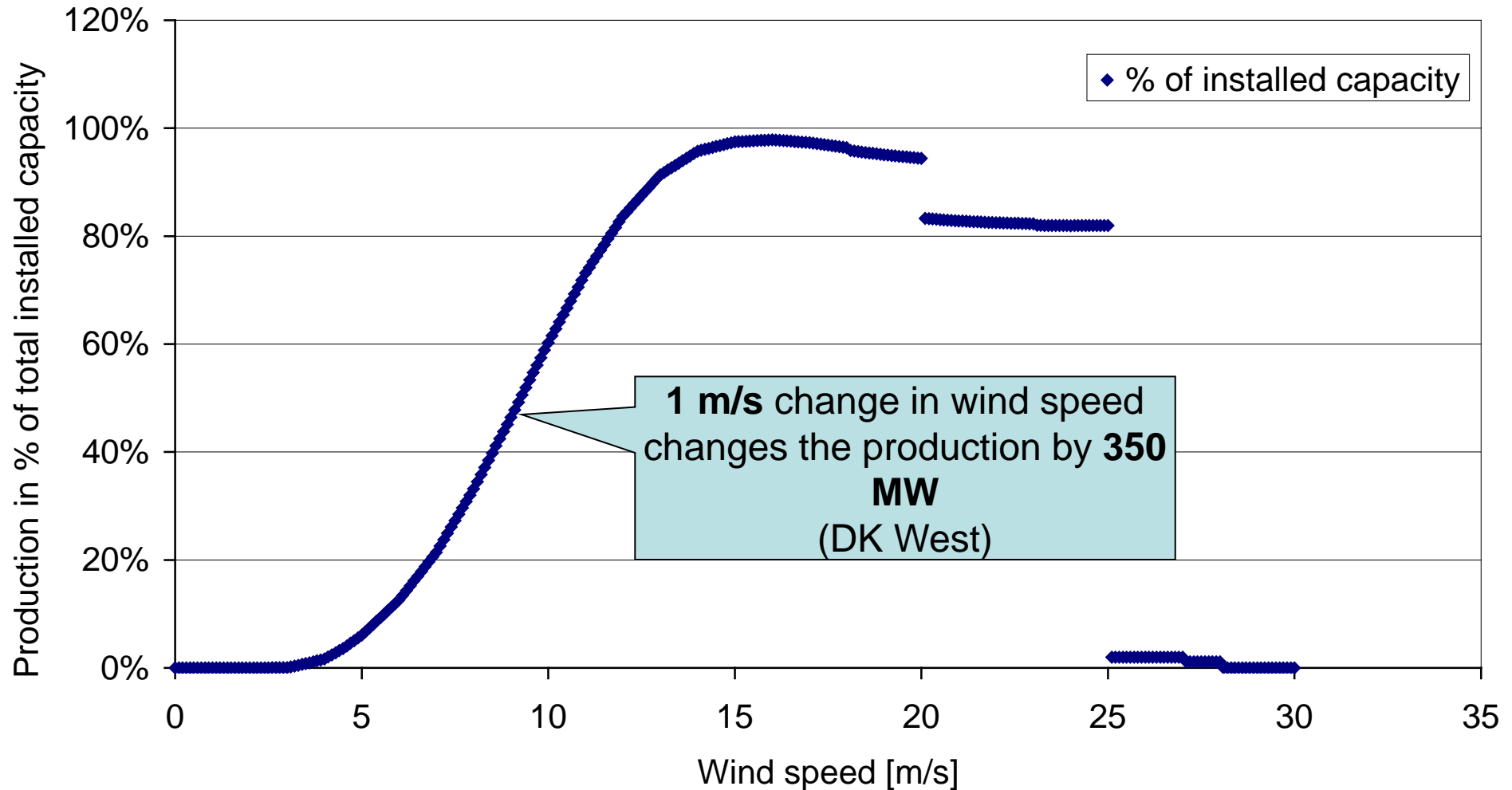
Smart Energy Systems



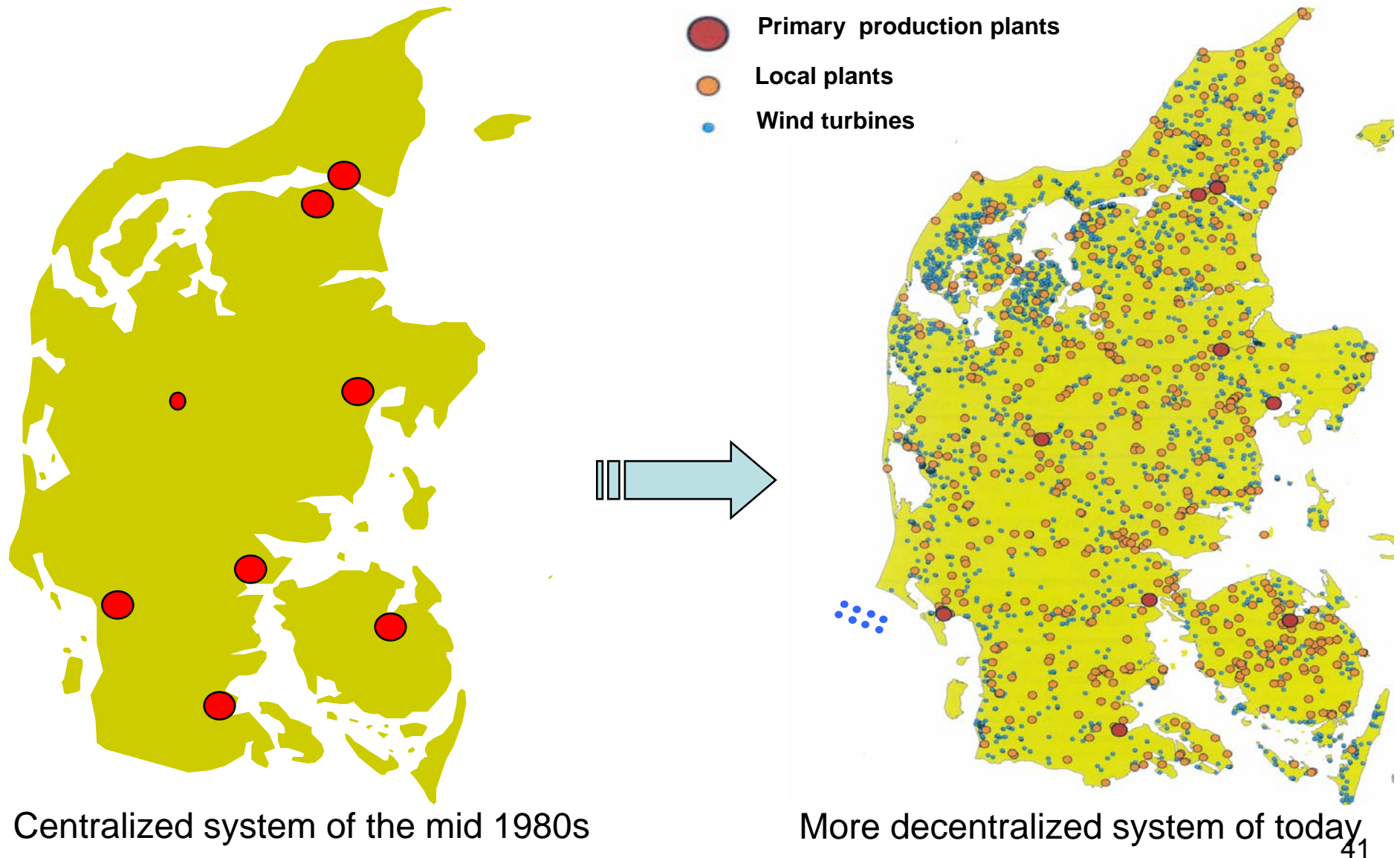
Wind Power



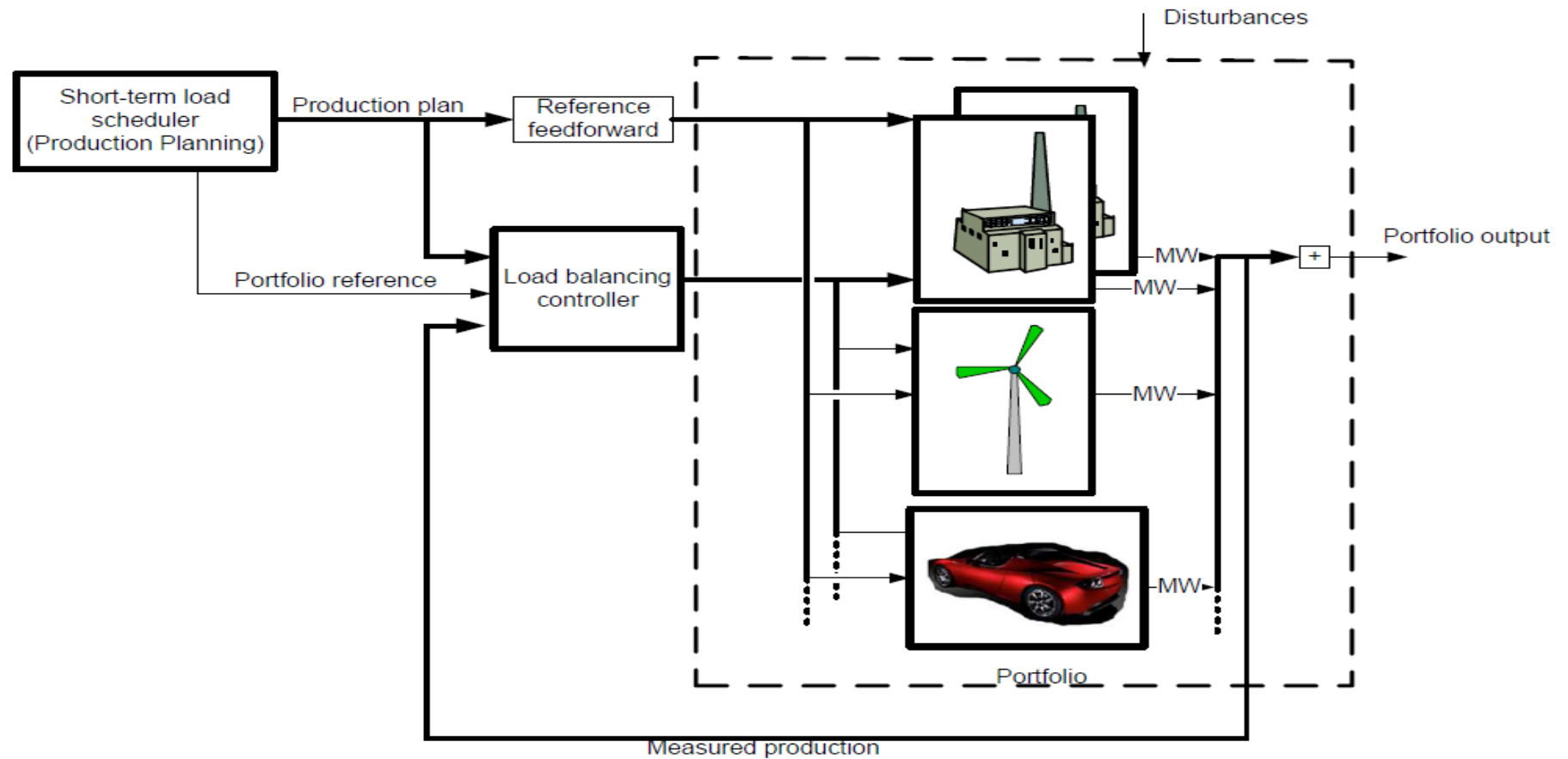
Wind Power Production and Imbalances



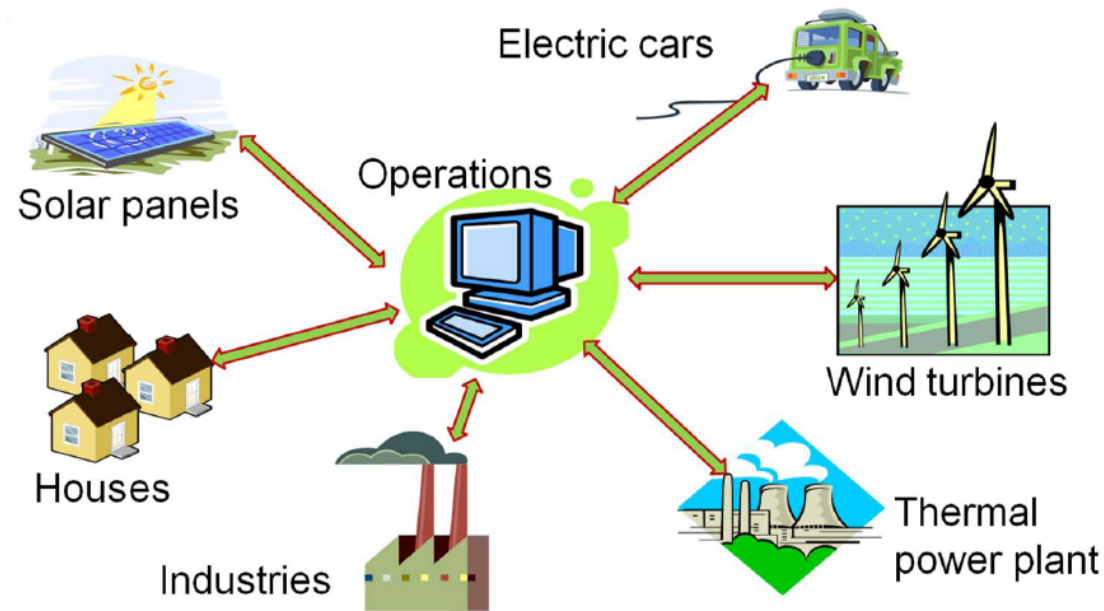
Power System Development – DK West



Load Balancing Controller



Models of Energy Components



$$\begin{aligned}x_{k+1}^i &= A_i x_k^i + B_i u_k^i + E_i d_k^i \\ y_k^i &= C_i x_k^i\end{aligned}$$

Controllable Power Generators

Each controllable power generator has an allowable operating range

$$u_{\min} \leq u_k \leq u_{\max}$$

and limits on how fast you can change the desired production

$$\Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max}$$

The unit production cost for power plant #i is: c_i .

Therefore, the total production cost over a given period is

$$\phi = \sum_{k=0}^{N-1} \sum_{i=1}^{n_u} c_i(u_k)_i = \sum_{k=0}^{N-1} c' u_k$$

MPC for Economic Power Portfolio Optimization

The portfolio power generation problem can be stated as

$$\begin{aligned} \min_{\{u_k\}_{k=0}^{N-1}} \quad & \phi = \sum_{k=0}^{N-1} c' u_k \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k \quad k = 0, 1, \dots, N-1 \\ & y_k = Cx_k \quad k = 1, 2, \dots, N \\ & u_{\min} \leq u_k \leq u_{\max} \quad k = 0, 1, \dots, N-1 \\ & \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} \quad k = 0, 1, \dots, N-1 \\ & y_k \geq r_k \quad k = 1, 2, \dots, N \end{aligned}$$

The parameters for this problem are

- Initial state, x_0 , and previous decision, u_{-1}
- Predicted loads on non-controllable generators (e.g. wind speed on wind turbines): $\{d_k\}_{k=0}^{N-1}$
- Predicted power demand: $\{r_k\}_{k=1}^N$

Soft Economic MPC

It may not always be possible to meet the power demand. Therefore, we relax the MPC problem

$$\begin{aligned}
 \min_{\{u_k, v_{k+1}\}_{k=0}^{N-1}} \quad & \phi = \sum_{k=1}^N \rho v_k + \sum_{k=0}^{N-1} c'_k u_k \\
 \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k \quad k = 0, 1, \dots, N-1 \\
 & y_k = Cx_k \quad k = 1, 2, \dots, N \\
 & u_{\min} \leq u_k \leq u_{\max} \quad k = 0, 1, \dots, N-1 \\
 & \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} \quad k = 0, 1, \dots, N-1 \\
 & y_k \geq r_k - v_k \quad k = 1, 2, \dots, N \\
 & v_k \geq 0 \quad k = 1, 2, \dots, N
 \end{aligned}$$

by introduction of the slack variables, v_k . ρ is selected sufficiently large, such that the power demand is met whenever possible.

Simple Test Example

Power Generator #1: Slow and Cheap

$$Y_1(s) = \frac{1}{(\tau_1 s + 1)^3} U_1(s) \quad \tau_1 = 20 \quad c_1 = 1$$

$$\begin{aligned} 0 &\leq u_k \leq 10 \\ -1 &\leq \Delta u_k \leq 1 \end{aligned}$$

Power Generator #2: Fast and Expensive

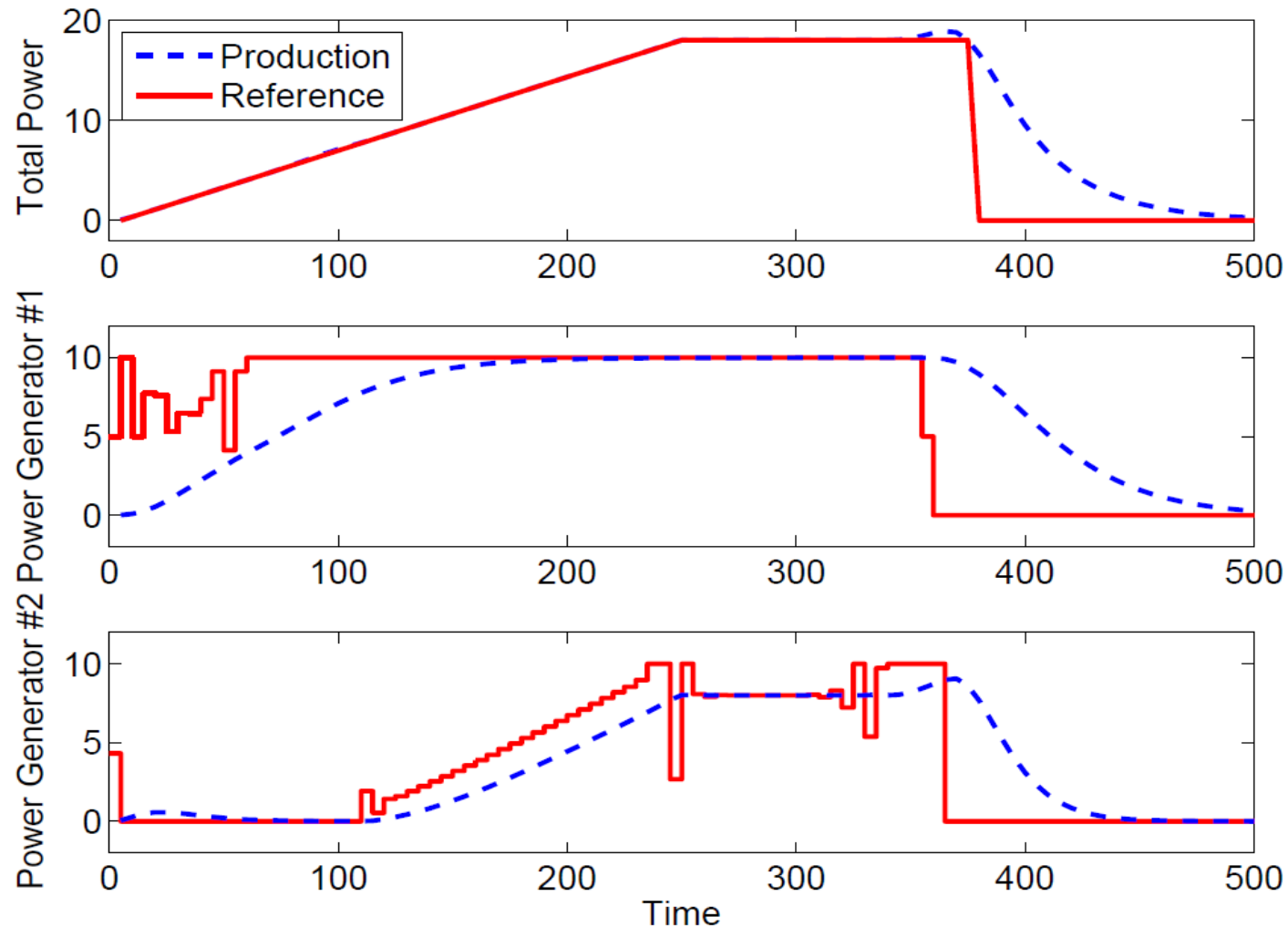
$$Y_2(s) = \frac{1}{(\tau_2 s + 1)^3} U_2(s) \quad \tau_2 = 10 \quad c_2 = 2$$

$$\begin{aligned} 0 &\leq u_k \leq 10 \\ -3 &\leq \Delta u_k \leq 3 \end{aligned}$$

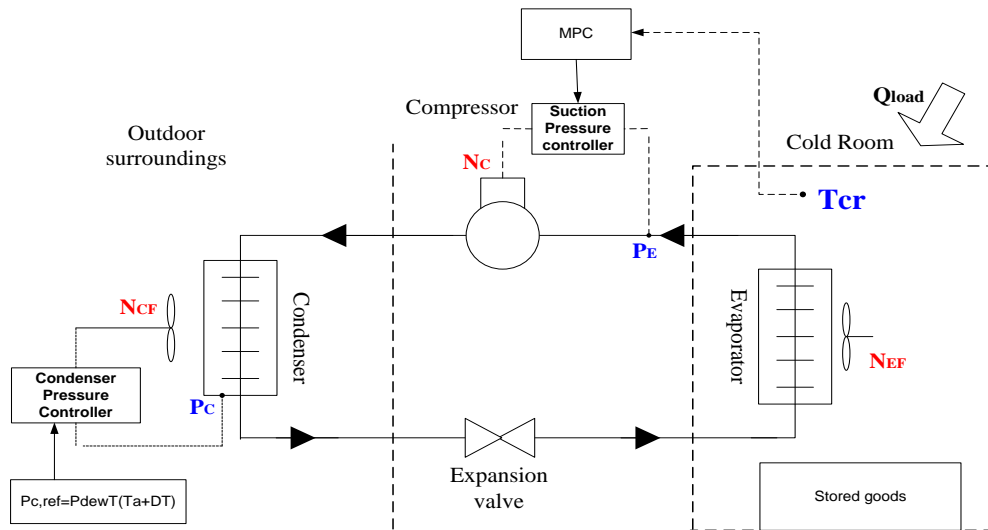
Total production: $Y(s) = Y_1(s) + Y_2(s)$

Penalty for producing less than sold: $\rho = 1.0 \cdot 10^5$

Optimal Production Profiles



Cooling Houses – A Flexible Consumer



Cooling system used in refrigerators and cooling houses

Danfoss

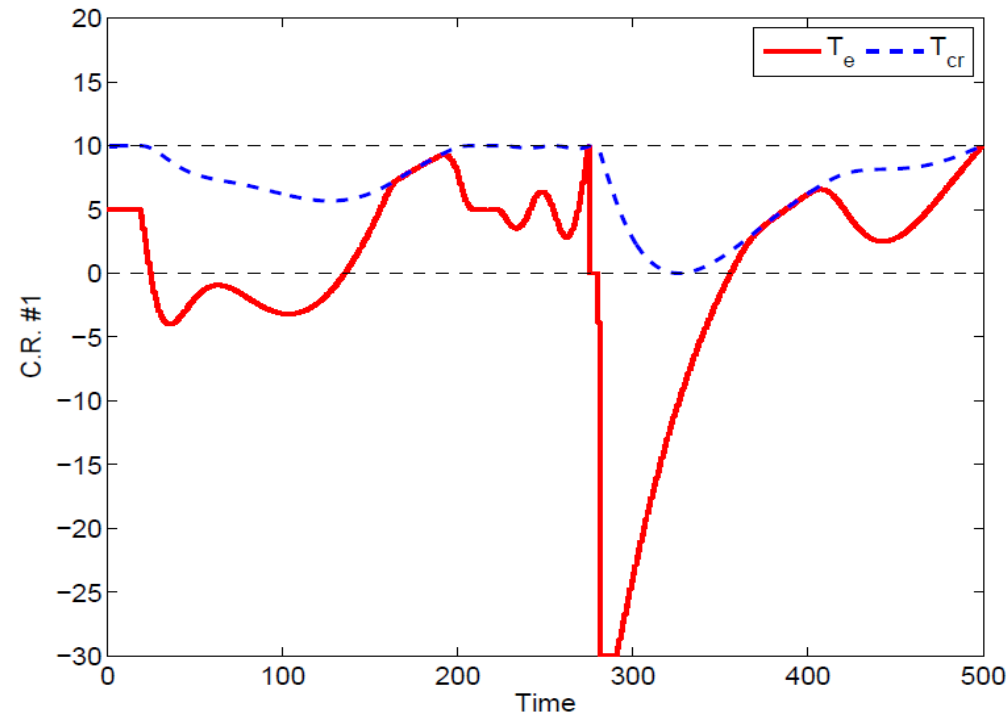
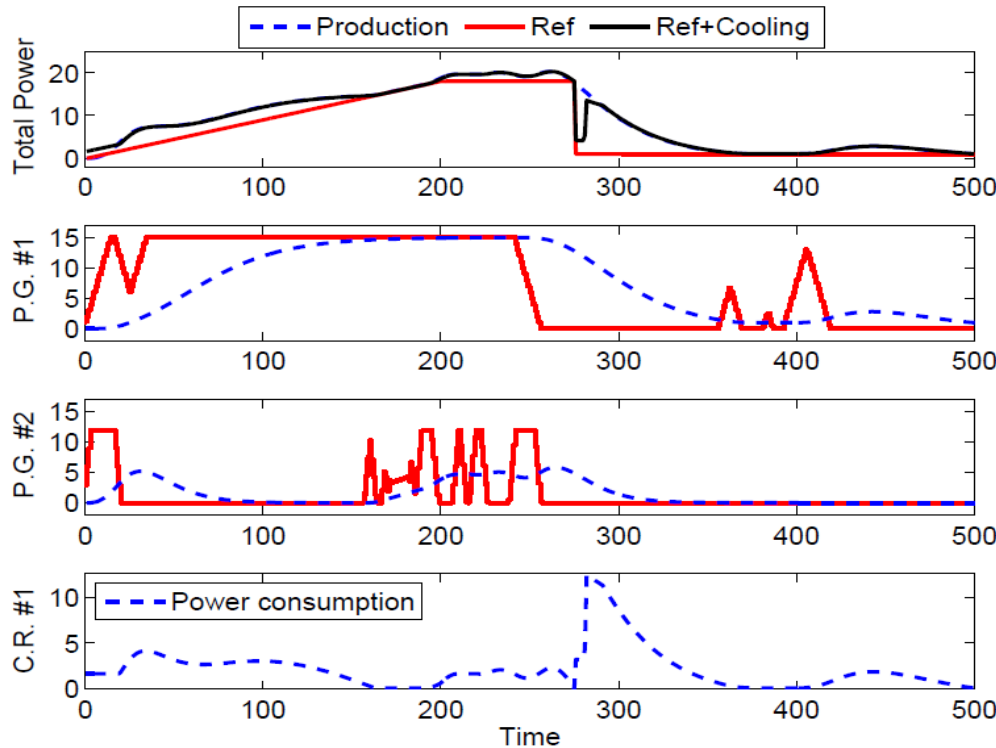
- **Motivation:**
- Refrigeration and air-conditioning consume substantial amounts of energy.
- E.g. up to 80% of energy consumed by supermarkets goes to refrigeration.
- **Methods:**
- Economic MPC:
Minimize the cost of cooling subject to temperature limits
- Predictions of weather, energy prices and load profiles.
- Implementation on industrial hardware.



FPGA

Power Management

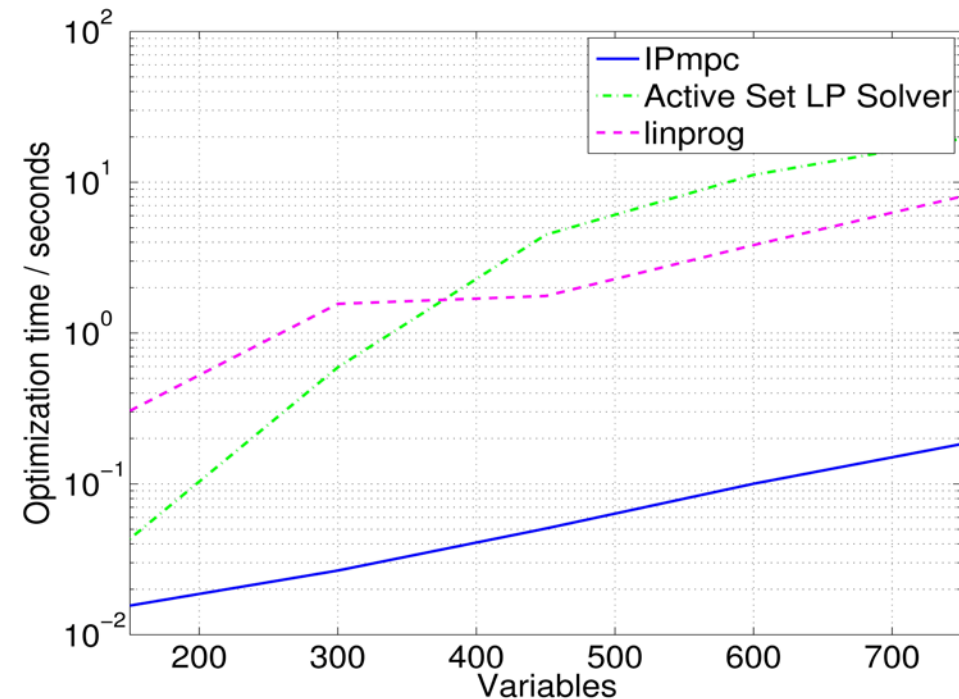
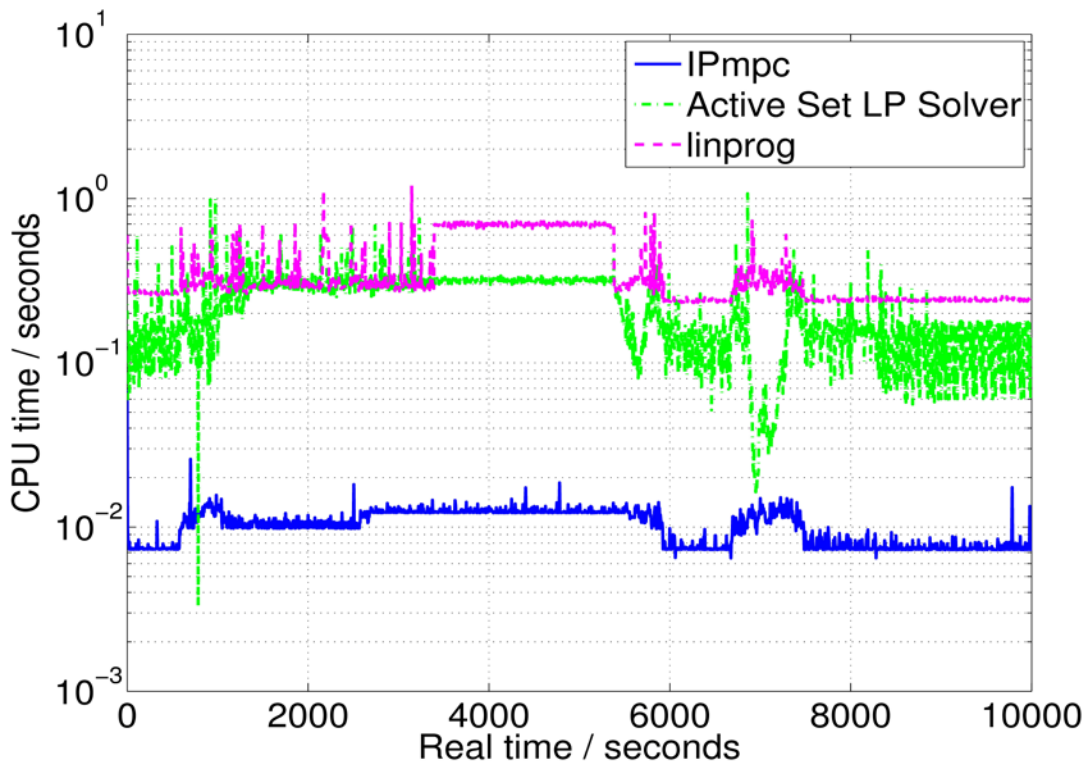
2 Power Plants and 1 Cooling House



- Economic Optimizing MPC
- Minimize power consumption and costs without lowering the cooling quality
- Load shifting utilizing the thermal capacity of the cooling house

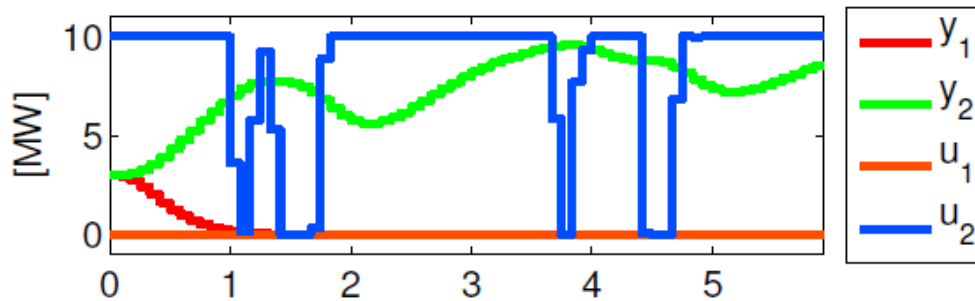
Online Computing Time

Tailored Primal-Dual Interior Point Algorithm

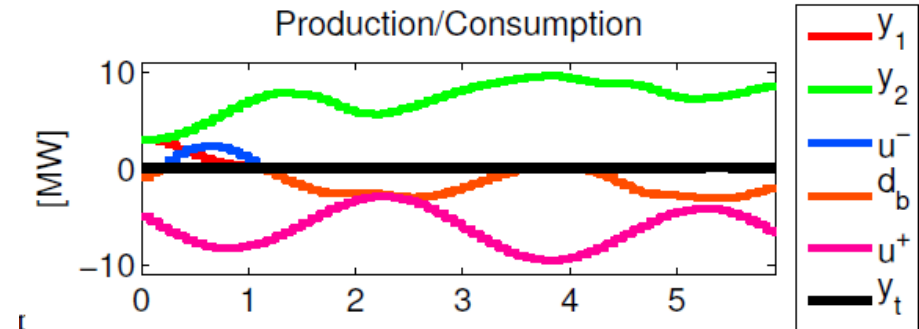


Warm Start Interior Point Methods

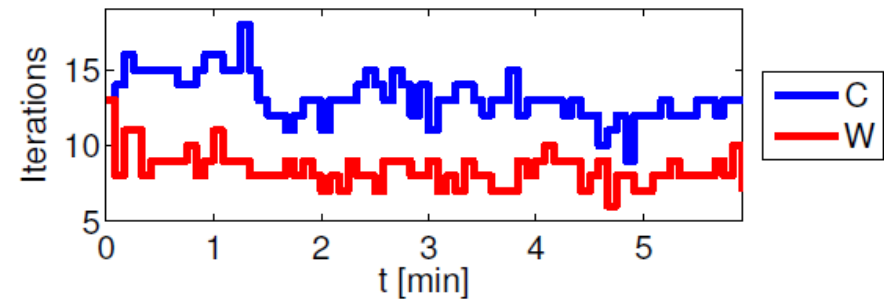
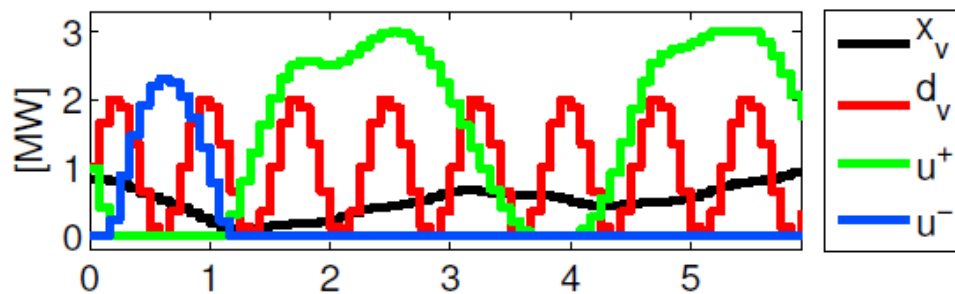
Power plant production and set points



Production/Consumption



EV charge / discharge



Accelerating LP Algorithms with GPUs

- Interior Point Methods are used for solving mathematical optimization problems eg:
 - While not converged
 - Form Hessian matrix: $H = A' DA$
 - Factorize Hessian: $L = chol(H)$
 - Compute affine step
 - Compute corrector step
 - Take step
 - Compute residuals
- Basic direct algorithm:
 - While not converged
 - Form Hessian matrix: $H = A' DA$
 - Factorize Hessian: $L = chol(H)$
 - Compute affine step
 - Compute corrector step
 - Take step
 - Compute residuals
- The main workload is the formation of the Hessian followed by the factorization.
- GPUs are very good at matrix-multiplication.

$$\begin{array}{ll} \min & \phi = g'x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{array}$$

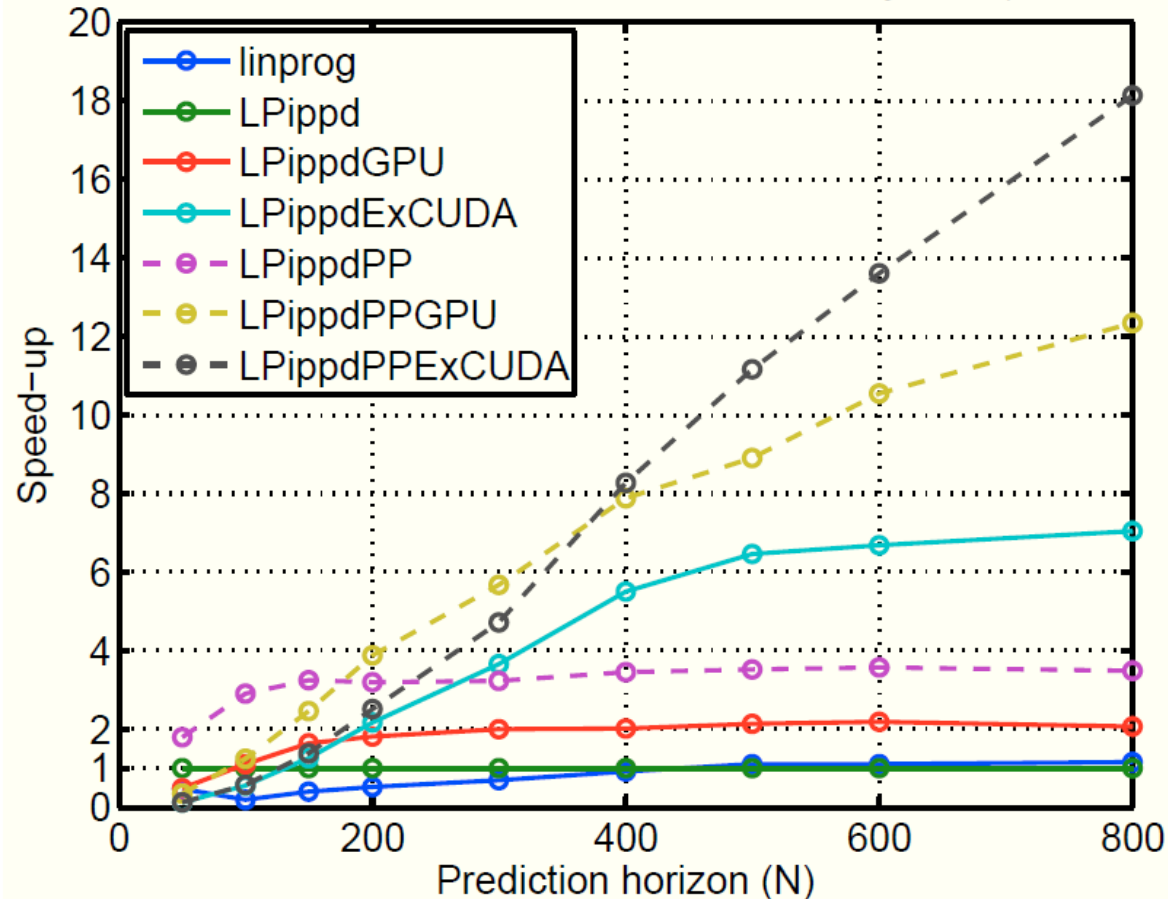
Implementation Details

- MATLAB CPU implementation as reference: LPippd
- MATLAB GPU implementation (MATLAB R2011b): LPippdGPU
- C + CUDA implementation: LPippdExCUDA
 - Uses available libraries for standard operations.
 - CUBLAS used for matrix operations.
 - MAGMA used for Cholesky factorization.
 - CUDA kernels implemented for additional operations such as computing step lengths.

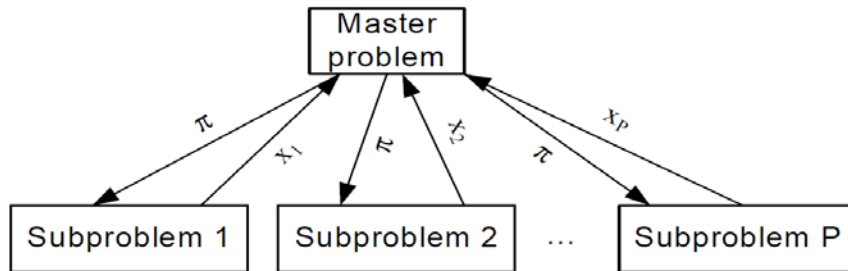


Speed-Up with GPU

LP Interior Point Method for the two Power Plant system (Tesla C2070)

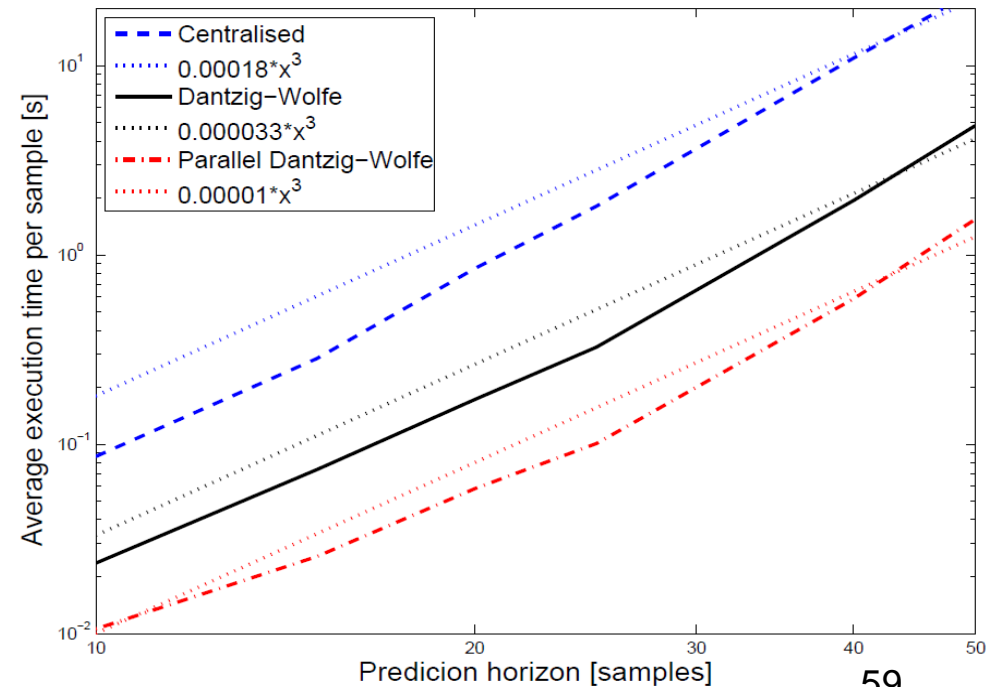
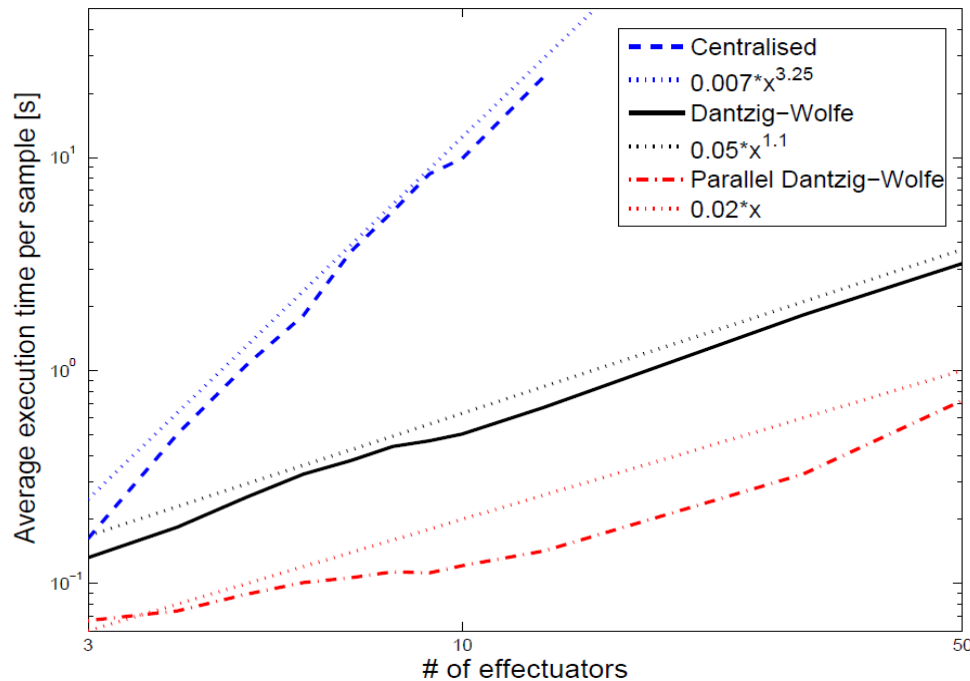


Dantzig-Wolfe Decomposition



$$\min_{\mathbf{z}_i} \quad \phi_i = [\mathbf{c}_i - \mathbf{F}'_i \pi]' \mathbf{z}_i$$

$$s.t. \quad \mathbf{G}_i \mathbf{z}_i \geq \mathbf{h}_i$$



Uncertainty

Smart energy systems are subject to uncertainty:

- Demand / Price / Weather



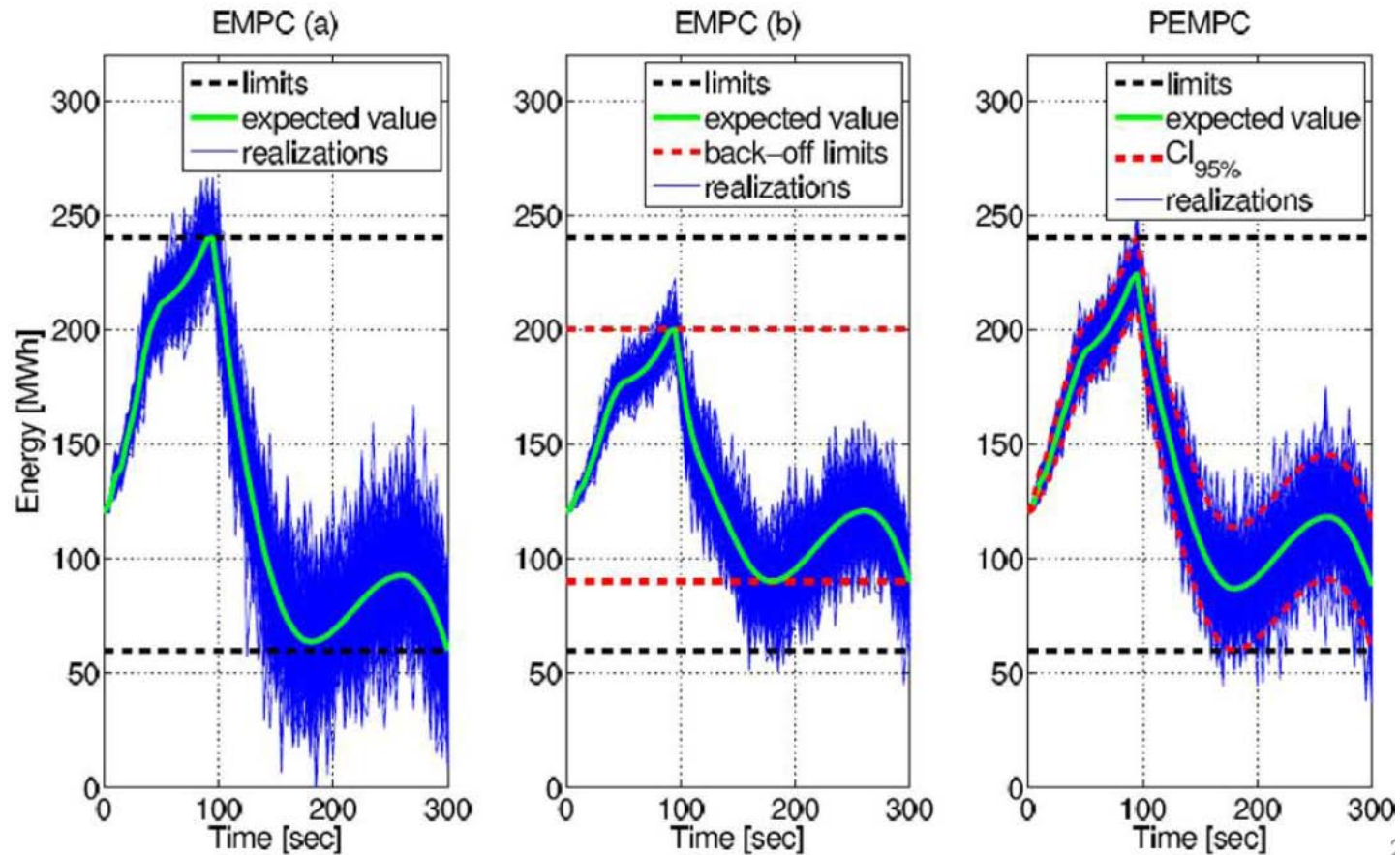
- Uncertainty on g and $b \implies$ LP

$$\begin{aligned} \min_x \mathbf{g}^T x \\ \text{s.t. } Ax \geq \mathbf{b} \quad j = 1, \dots, m \\ x \in \mathbb{R}^n \end{aligned}$$

- Uncertainty on g , A and b with Gaussian distribution \implies SOCP

$$\begin{aligned} \min_x \mathbf{g}^T x \\ \text{s.t. } \mathbf{A}_{[j,:]} x \geq \mathbf{b}_j \quad j = 1, \dots, m \\ x \in \mathbb{R}^n \end{aligned} \implies \begin{aligned} \min_x g^T x \\ \text{s.t. } Ax \geq b \quad j = 1, \dots, m \\ x \succeq_{K^n} 0 \end{aligned}$$

Uncertainty Scenarios



The Extended LQ Problem

$$\min_{\{u_k, x_{k+1}\}_{k=0}^{N-1}} \phi = \sum_{k=0}^{N-1} l_k(x_k, u_k) + l_N(x_N)$$

$$s.t. \quad x_{k+1} = A_k x_k + B_k u_k + b_k \quad k \in \mathcal{N}$$

with $\mathcal{N} = \{0, 1, \dots, N-1\}$ and stage costs defined by

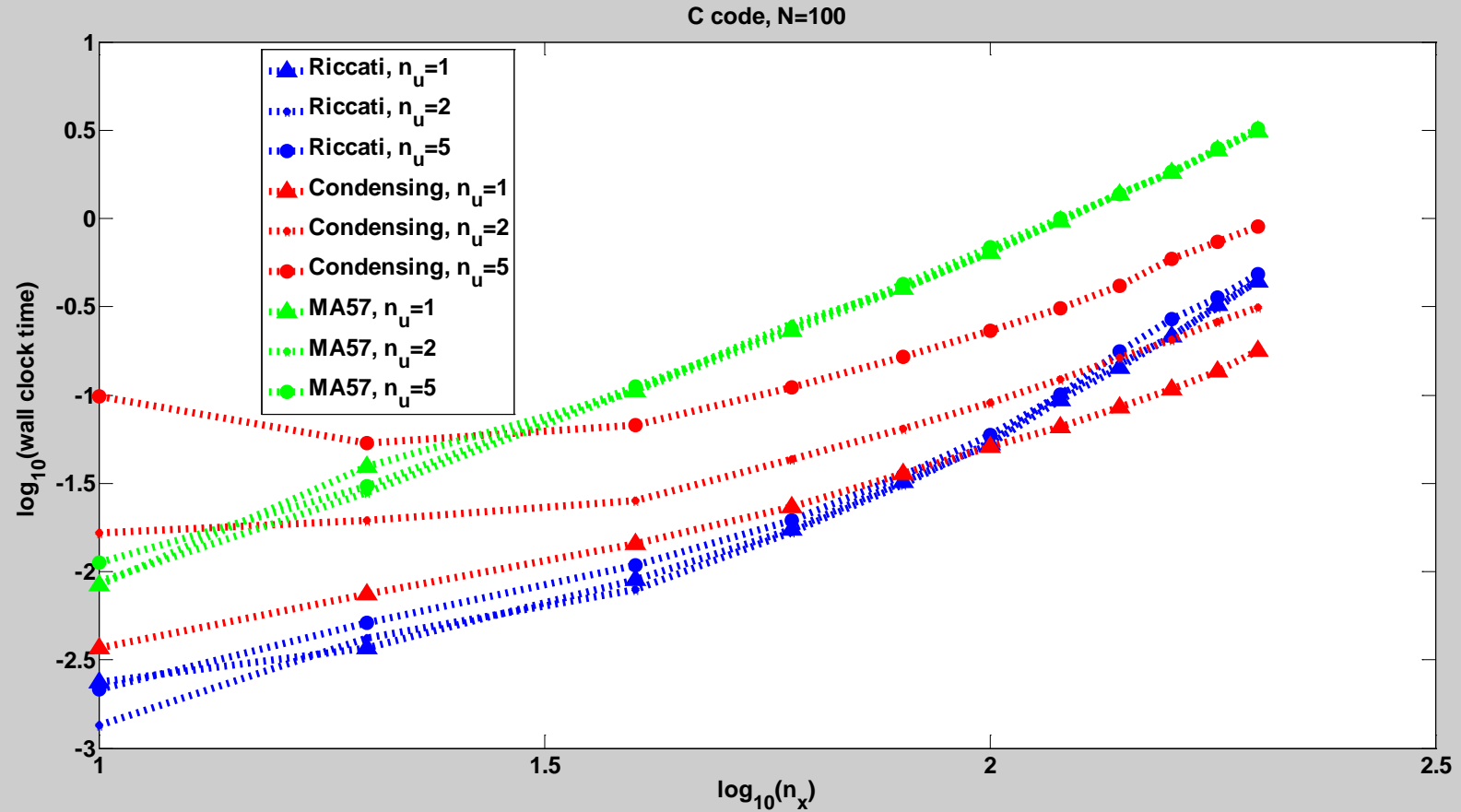
$$l_k(x_k, u_k) = \frac{1}{2} \begin{bmatrix} x_k \\ u_k \end{bmatrix}' \begin{bmatrix} Q_k & M_k' \\ M_k & R_k \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \begin{bmatrix} q_k \\ s_k \end{bmatrix}' \begin{bmatrix} x_k \\ u_k \end{bmatrix} + \rho_k$$

$$l_N(x_N) = \frac{1}{2} x_N' P_N x_N + p_N' x_N + \gamma_N$$

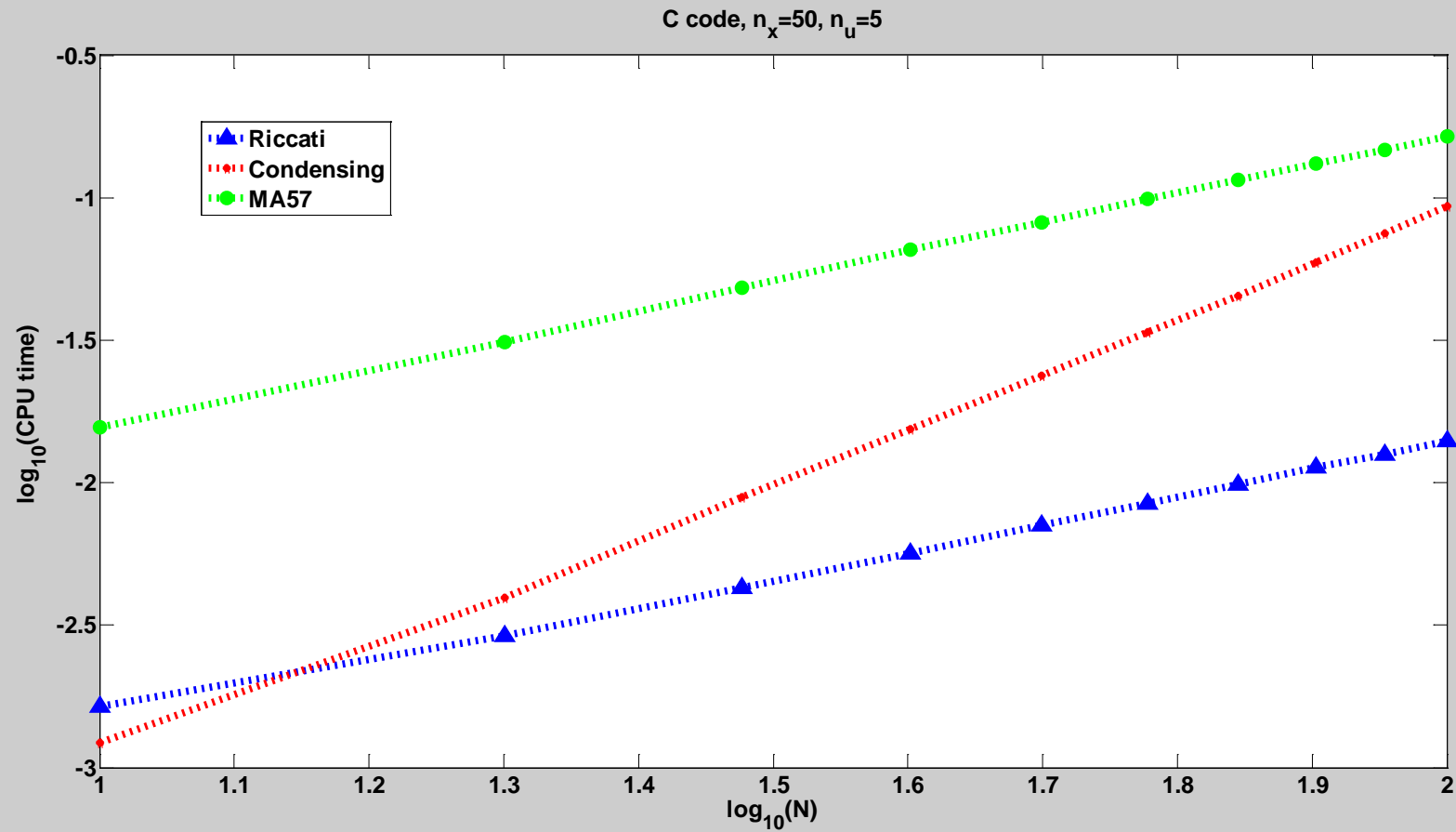
KKT System for the Extended LQ Problem

$$\left[\begin{array}{cc|ccc|cc|c} R_0 & B_0' & & & & & & \\ B_0 & 0 & -I & & & & & \\ \hline & -I & Q_1 & M_1' & A_1' & & & \\ & & M_1 & R_1 & B_1' & & & \\ & & A_1 & B_1 & 0 & -I & & \\ \hline & & & & & -I & Q_2 & M_2' & A_2' \\ & & & & & & M_2 & R_2 & B_2' \\ & & & & & & A_2 & B_2 & 0 & -I \\ \hline & & & & & & & & & -I & P_3 \end{array} \right] \begin{bmatrix} u_0 \\ \pi_1 \\ x_1 \\ u_1 \\ \pi_2 \\ x_2 \\ u_2 \\ \pi_3 \\ x_3 \end{bmatrix} = - \begin{bmatrix} \tilde{s}_0 \\ \tilde{b}_0 \\ q_1 \\ s_1 \\ b_1 \\ q_2 \\ s_2 \\ b_2 \\ p_3 \end{bmatrix}$$

Numerical Results



Numerical Results



Summary

- There is a need for HPC software supporting MPC
 - QP, LP, SOCP algorithms
 - ODE/PDE solvers equipped with sensitivity computing abilities (adjoints)
 - SQP / NLP algorithms
 - Efficient direct sparse and iterative methods for KKT systems (even when they are ill-conditioned)
- The number of potential applications is very large

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